BE-Alg.2 | WEDNESDAY 10-28-09

ACT

Barrons

1. WHAT IS THE MATRIX PRODUCT?

$$\begin{bmatrix} x & 2x & 4x \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$$

ANS)

$1 \times 3 \cdot 3 \times 1$ ✓ OK

$3 = 3$ ✓

PRODUCT MATRIX IS 1×1

$-2 \quad 0 \quad 5$

$$\begin{bmatrix} x & 2x & 4x \end{bmatrix} \longrightarrow \begin{bmatrix} -2x + 0 + 20x \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{bmatrix} 18x \end{bmatrix}} \longleftarrow \text{ANS}$$

Using Real Numbers Only:

$$\text{Real} = \begin{cases} \text{RATIONAL} \Rightarrow \{\text{terminating decimals, Repeating decimals}\} & \text{FRACTION NUMBERS} \\ \text{IRRATIONAL} \Rightarrow \{\text{NON-repeating infinite decimals}\} & \text{NOT FRACTION NUMBERS} \end{cases}$$

Why does $\sqrt{-16}$ NOT EXIST?

$(-4)(-4) \neq -16$ AND $(4)(4) \neq -16$

Since $\sqrt{-16} = \underbrace{\sqrt{-1}}_{= i}\sqrt{16}$ $\boxed{\text{DEFINE: } \sqrt{-1} = i}$

$\therefore \sqrt{-16} = i\,4 = 4i$

imaginary number

REALS + imaginary

4 — pure real

$4i$ — pure imaginary

$$\underbrace{4}_{\text{Real part}} + \underbrace{4i}_{\text{imaginary part}}$$

$\boxed{\text{COMPLEX NUMBERS}} = \{\text{Reals, imaginary}\}$

Ex $\sqrt{-25} = 5i$

$\sqrt{-18} = \sqrt{9}\sqrt{2}\, i = 3\sqrt{2}\, i = \boxed{3i\sqrt{2}}$

$\sqrt{-1} = i$

WHY PREFERRED?

Multiplying Imaginary Numbers:

Ex 1 pg 270 → Ch. 5-9 Complex Numbers

A. $-2i \cdot 7i = -14i^2 = -14(-1) = \boxed{14}$

WHY?

* $i = \sqrt{-1} = i$

$i^2 = (\sqrt{-1})^2 = -1$

$i^3 = i^2 i = -i$

$i^4 = i^2 i^2 = +1$

Know This!

* $i^4 = (i^2)^2 = (-1)^2 = 1$

B. $\sqrt{-10} \cdot \sqrt{-15} = (i\sqrt{10})(i\sqrt{15}) = i^2\sqrt{150}$

$(-1)\sqrt{25}\sqrt{6}$

$= -1 \cdot 5 \cdot \sqrt{6}$

$= \boxed{-5\sqrt{6}}$

$\Rightarrow \sqrt{(-10)\cdot(-15)}$

$\sqrt{-1} \cdot \sqrt{10} \cdot \sqrt{-1} \cdot \sqrt{15}$

If $\sqrt{-1}$ EXISTS $\Rightarrow -1 \cdot \sqrt{150} = -5\sqrt{6}$

Writing Real Numbers as Complex and Vice Versa...

$$12 = 12 + 0i$$

$$8i = 0 + 8i$$

$$6 + 5i = 6 + 5i$$

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powers of i

Ex 3 Pg 270

$$i^{45} = ?$$

$$i^1 \cdot i^{44} = i\,(i^2)^{22}$$

$$i\,(-1)^{22} = \boxed{i}$$

Ex

$$i^{58} = ?$$

$$i^1\, i^{57}$$

$$i\,[i^1 \cdot i^{56}]$$

$$i\,[i \cdot (i^2)^{54}]$$

$$i\,[i \cdot (-1)^{54}]$$

$$i\,[i] = i^2 = \boxed{-1}$$

SHORTCUT

$$i^N = i^R$$

R = Remainder of $\frac{N}{4}$

$$i^{58} \Rightarrow 4\overline{)58} = 14 \text{ R } 2$$

$$\begin{array}{r} 4 \\ \hline 18 \\ 16 \\ \hline 2 \end{array}$$

$$i^{58} = i^2 = \boxed{-1}$$ ✓

Solving and checking solutions to equations when the solutions are complex:

Ex4 Pg271 Solve $3x^2 + 48 = 0$

$$3x^2 = -48$$

$$x^2 = -16$$

$$x = \pm\sqrt{-16} = \boxed{\pm 4i}$$

CK $4i$

$$3(\ )^2 + 48 \overset{?}{=} 0$$

$$3(4i)^2 + 48 \overset{?}{=} 0$$

$$3 \cdot 16 \cdot i^2 + 48 \overset{?}{=} 0$$

$$48(-1) + 48 = 0 \ \checkmark$$

CK $-4i$

$$3(-4i)^2 + 48 \overset{?}{=} 0$$

$$3 \cdot 16 i^2 + 48 \overset{?}{=} 0 \ \checkmark$$ Since same as above

OR

$$48i^2 + 48 \overset{?}{=} 0$$

(Finish)

$$48(-1) + 48 = 0 \ \checkmark$$

Property of Equality:

Two complex numbers are equal iff the real parts are equal and the imaginary parts are equal.

(Ex) $2x + 5i = 2x + 5i$

Ex 5 Pg 271 If $\underbrace{2x - 3}_{\text{Real}} + \underbrace{(y - 4)}_{\text{imag.}} i = \underbrace{3}_{} + \underbrace{2}_{\text{imag}} i$

Then

$2x - 3 = 3 \quad \therefore 2x = 6 \quad \boxed{x = 3}$

and $y - 4 = 2 \quad \therefore \boxed{y = 6}$

To add and subtract complex numbers

$\Rightarrow$ add real parts, add complex parts

Ex 6 Pg 272

(A) $(6 - 4i) + (1 + 3i) = \boxed{7 - i}$

(B) $(3 - 2i) - (5 - 4i) = \boxed{-2 + 2i}$

Multiplying Complex Numbers
⇒ Like Multiplying Binomials = "FOIL"

EX 7 PG 273 $(1+3i)(7-5i)$

$7-5i+21i-15i^2$

$7+16i-15(-1)$

$7+16i+15 = \boxed{22+16i}$

Note: In division ⇒ don't leave i in bottom.

Ex $\frac{1}{4i}\cdot\frac{4i}{4i} = \frac{4i}{-16} = \frac{-i}{4}$

Ex $\frac{1}{2+3i}\cdot\frac{2-3i}{2-3i} = \frac{2-3i}{4-9(-1)} = \frac{2-3i}{13}$

Homework: Pg. 273 # 4-11
Pg 274 # 48-51