BE - Alg. 2 | Tuesday 11-10-09

Solve by Completing the Square:

(1.) $\frac{3x^2}{3} - \frac{5x}{3} + \frac{1}{3} = \frac{0}{3}$

$$x^2 - \frac{5}{3}x + \{\ \} = -\frac{1}{3} + \{\ \}$$

$$x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{12}{36} + \frac{25}{36} = \frac{13}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{13}{36}$$

$$x - \frac{5}{6} = \pm\sqrt{\frac{13}{36}} = \pm\frac{\sqrt{13}}{\sqrt{36}} = \frac{\pm\sqrt{13}}{6}$$

$$x = \frac{5}{6} \pm \frac{\sqrt{13}}{6} = \boxed{\frac{5 \pm \sqrt{13}}{6} = x}$$

Ck $3\left(\frac{5+\sqrt{13}}{6}\right)^2 - 5\left(\frac{5+\sqrt{13}}{6}\right) + 1 \stackrel{?}{=} 0$

$$\cancel{3}^{1}\left[\frac{25+10\sqrt{13}+13}{\cancel{36}_{12}}\right] - \left[\frac{25+5\sqrt{13}}{6}\right] + 1 \stackrel{?}{=} 0$$

$$\frac{38+10\sqrt{13}}{12} - \left[\frac{25+5\sqrt{13}}{6}\right] + 1 \stackrel{?}{=} 0$$

$$\frac{19+5\sqrt{13}}{6} - \left[\frac{25+5\sqrt{13}}{6}\right] + 1 \stackrel{?}{=} 0$$

$$\frac{19}{6} + \frac{5\sqrt{13}}{6} - \frac{25}{6} - \frac{5\sqrt{13}}{6} + 1 \stackrel{?}{=} 0 \checkmark$$

Homework review Pg. 311 #32-36

(32) $x^2 - 8x + 15 = 0$

$x^2 - 8x + 16 = -15 + 16 = 1$

$(x-4)^2 = 1$

$x - 4 = \pm\sqrt{1} = \pm 1 \quad \therefore \boxed{x = \{5, 3\}}$

Ck: $(5)^2 - 8(5) + 15 \stackrel{?}{=} 0$ ✓ Ck: $(3)^2 - 8(3) + 15 \stackrel{?}{=} 0$ ✓

---

(33) $x^2 + 2x - 120 = 0$

$x^2 + 2x + 1 = 120 + 1$

$(x+1)^2 = 121$

$x + 1 = \pm\sqrt{121} = \pm 11$

$x = -1 \pm 11 \quad \therefore \boxed{x = \{10, -12\}}$

Ck $(10)^2 + 2(10) - 120 \stackrel{?}{=} 0$

$100 + 20 - 120 \stackrel{?}{=} 0$ ✓

Ck $(-12)^2 + 2(-12) - 120 = 0$

$144 - 24 - 120 \stackrel{?}{=} 0$ ✓

(34) $x^2 + 2x - 6 = 0$

$x^2 + 2x + \{1\} = 6 + \{1\}$

$(x+1)^2 = 7$

$x + 1 = \pm\sqrt{7} \quad \therefore \boxed{x = -1 \pm \sqrt{7}}$

$\underline{\underline{\text{CK}}}$ $(-1+\sqrt{7})^2 + 2(-1+\sqrt{7}) - 6 \stackrel{?}{=} 0$

$\overbrace{1 - 2\sqrt{7} + 7} - 2 + 2\sqrt{7} - 6 \stackrel{?}{=} 0$

$8 - 8 \stackrel{?}{=} 0$ ✓

$\underline{\underline{\text{CK}}}$ $(-1-\sqrt{7})^2 + 2(-1-\sqrt{7}) - 6 \stackrel{?}{=} 0$

$1 + 2\sqrt{7} + 7 - 2 - 2\sqrt{7} - 6 \stackrel{?}{=} 0$

$8 - 8 \stackrel{?}{=} 0$ ✓

---

(35) $x^2 - 4x + 1 = 0$

$x^2 - 4x + \{4\} = -1 + \{4\} = 3$

$(x-2)^2 = 3$

$x - 2 = \pm\sqrt{3} \quad \therefore \boxed{x = 2 \pm \sqrt{3}}$

$\underline{\underline{\text{CK}}}$ $(2+\sqrt{3})^2 - 4(2+\sqrt{3}) + 1 \stackrel{?}{=} 0$

$4 + 4\sqrt{3} + 3 - 8 - 4\sqrt{3} + 1 \stackrel{?}{=} 0$ ✓

$\underline{\underline{\text{CK}}}$ $(2-\sqrt{3})^2 - 4(2-\sqrt{3}) + 1 \stackrel{?}{=} 0$

$4 - 4\sqrt{3} + 3 - 8 + 4\sqrt{3} + 1 \stackrel{?}{=} 0$ ✓

36 $x^2 - 4x + 5 = 0$

$x^2 - 4x + 4 = -5 + 4$

$(x-2)^2 = -1$

$x - 2 = \pm\sqrt{-1} = \pm i$

$\boxed{x = 2 \pm i}$

Ck $(2+i)^2 - 4(2+i) + 5 \stackrel{?}{=} 0$

$4 + \underline{4i} + i^2 - 8 \underline{-4i} + 5 \stackrel{?}{=} 0$

$4 \quad -1 \quad -8 \quad +5 \stackrel{?}{=} 0$ ✓

Ck $(2-i)^2 - 4(2-i) + 5 \stackrel{?}{=} 0$

$4 \underline{-4i} + i^2 - 8 \underline{+4i} + 5 \stackrel{?}{=} 0$

$4 \quad -1 \quad -8 \quad +5 \stackrel{?}{=} 0$ ✓

It would be nice to be able to tell if you could factor a quadratic (so you could solve it by factoring/ZPP). Otherwise you must Complete The Square.

There is a tool that tells you important facts about the quadratic. It is called the discriminant.

(Don't mix this up with the determinant of a matrix).

$d = b^2 - 4ac$ = discriminant

d = Perfect Square $\Rightarrow$ The solutions to the quadratic are rational and it can be factored

d = Perfect Square $= 0$ $\Rightarrow$ S/A but it has a "double root", ie, it is a Perfect Square Trinomial

Ex. # 32 of homework:

$$x^2 - 8x + 15$$

$a = 1$ $\quad b^2 - 4ac$

$b = -8$ $\quad (-8)^2 - 4(1)(15)$

$c = 15$ $\quad 64 - 60 = \boxed{4 = d}$ Perfect Square

- Rational roots
- Factorable

Ex. Try the Perfect Square Trinomial you created in # 32 by Completing The Square

$$x^2 - 8x + 16$$

$a = 1$ $\quad b^2 - 4ac$

$b = -8$ $\quad (-8)^2 - 4(1)(16)$

$c = 16$ $\quad 64 - 64 = \boxed{0 = d}$ Perfect Square

- rational double roots
- factorable, Perfect Square Trinomial

- Homework Pg. 311 # 37, 38, 39, 41
  Solve, plus find d & state which quadratics could be factored.