BE-Alg.2 Monday 11-16-09

Find the common denominator, then combine:

① $\frac{1}{x} + \frac{1}{y} = ?$

② $\frac{1}{x} + \frac{1}{x^2} = ?$ Do Not use x^3!

③ $\frac{-5}{x} + \frac{2y}{4x^2} = ?$ Do Not use $4x^3$!

Ans:

① $\frac{1}{x} + \frac{1}{y} \Rightarrow \frac{\quad}{xy} + \frac{\quad}{xy} = \frac{\quad}{xy}$

mult. x by what to get xy? mult. y by what to get xy?

$\frac{1}{x}\cdot\frac{y}{y} + \frac{1}{y}\cdot\frac{x}{x} = \frac{y}{xy} + \frac{x}{xy} = \boxed{\frac{y+x}{xy}}$

② $\frac{1}{x} + \frac{1}{x^2} \Rightarrow \frac{\quad}{x^2} + \frac{\quad}{x^2} = \frac{\quad}{x^2}$

$\frac{x}{x^2} + \frac{1}{x^2} = \boxed{\frac{x+1}{x^2}}$ Ck $\frac{x}{x^2} + \frac{1}{x^2}$ ✓

③ $\frac{-5}{4x^2}\cdot\frac{4x}{4x} + \frac{2y}{4x^2} = \frac{-20x + 2y}{4x^2}$

Complete the Square for $ax^2+bx+c=0$

NOTES

$$ax^2+bx+c=0$$

Standard Form, ANY quadratic

$$\frac{ax^2}{a}+\frac{bx}{a}=\frac{-c}{a}$$

Move "c", ÷ by a

$$x^2+\frac{b}{a}x+\square=-\frac{c}{a}+\square$$

$$\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2=\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$

Complete the Square term

$$x^2+\frac{b}{a}x+\left\{\frac{b^2}{4a^2}\right\}=-\frac{c}{a}+\frac{b^2}{4a^2}$$

Change $-\frac{c}{a}$ to common denominator of $4a^2$ $\left\{\frac{4a^2}{a}=4a\right\}$

$$\therefore \frac{-c}{a}\cdot\frac{4a}{4a}=\frac{-4ac}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}=\frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

THE QUADRATIC FORMULA

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}} \quad \text{or} \quad \boxed{x=\frac{-b\pm\sqrt{d}}{2a}}$$

Since $d=b^2-4ac$

d = discriminant

Ch. 6-4 The Quadratic Formula (and the Discriminant)

Ex 1 Pg 314 Solve — using the QF:

$$x^2 - 12x = 28$$

$$x^2 - 12x - 28 = 0$$

$a = 1$, $b = -12$, $c = -28$

$$b^2 - 4ac$$

$$(-12)^2 - 4(1)(-28)$$

$$144 + 112 = \boxed{256 = d}$$

$$x = \frac{-b \pm \sqrt{d}}{2a} = \frac{-(-12) \pm \sqrt{256}}{2(1)}$$

$$x = \frac{12 \pm 16}{2}$$

$$\boxed{x = \{14, -2\}}$$

Look: d = perfect square ⇒ factorable ⇒ 2 rational roots since $\sqrt{PS}$ is rational

$$\{(x-14)(x+2) = 0$$

Ck $(14)^2 - 12(14) \stackrel{?}{=} 28$

$196 - 168 \stackrel{?}{=} 28$ ✓

Ck $(-2)^2 - 12(-2) \stackrel{?}{=} 28$

$4 + 24 \stackrel{?}{=} 28$ ✓

ACT Nice to Know

144
169
196
225
256
289
324
361
400

WHAT ARE THE OTHER CONSEQUENCES of having d under the square root in the QF $\Rightarrow x = \frac{-b \pm \sqrt{d}}{2a}$?

d	Solution	Notes
Perfect Square	2 rational numbers	factorable
0	1 rational number	$\pm\sqrt{0} = 0$ $x = \frac{-b}{2a}$ factorable
Not a Perfect Square	2 irrational Numbers	(Ex) $\pm\sqrt{5}$ $\pm\sqrt{6}$ etc.
Negative	2 complex numbers	(Ex) $\pm 4i$ $\pm 3i\sqrt{6}$

Ex 2
Pg 314
QF

$x^2 + 22x + 121 = 0$

$a = 1$
$b = 22$
$c = 121$

$b^2 - 4ac$

$(22)^2 - 4(1)(121)$

$484 - 484 = \boxed{0 = d}$

$x = \frac{-b \pm \sqrt{d}}{2a} = \frac{-22 \pm \sqrt{0}}{2(1)}$

$x = \frac{-22}{2} \Rightarrow \boxed{x = \{-11\}}$

CK $(-11)^2 + 22(-11) + 121 \stackrel{?}{=} 0$

$121 - 242 + 121 \stackrel{?}{=} 0$ ✓

Since $d = 0$, must have a Perfect Sq. Trinomial

$\Rightarrow x^2 + 22x + 121 = 0$

$2 \cdot x \cdot 11$ ✓

$(x + 11)^2 = 0$

$x + 11 = 0$

$x = -11$ ✓

- Keep an eye out for a GCF
- Always put in ax^2+bx+c form.

EX3 PG315

$2x^2+4x-5=0$

$a=2$, $b=4$, $c=-5$

b^2-4ac

$(4)^2-4(2)(-5)$

$16+40 = \boxed{56=d}$

$\sqrt{56}$
$\sqrt{4}\sqrt{14}$

$x=\frac{-b\pm\sqrt{d}}{2a}=\frac{-4\pm\sqrt{56}}{2(2)}$

$x=\frac{-4\pm 2\sqrt{14}}{4}$

·1
·$\frac{1}{2}$
·$\frac{1}{2}$

÷ All 3 terms by 2

$\boxed{x=\frac{-2\pm\sqrt{14}}{2}}$

d was ⊕ but NOT a perfect square ⇒ 2 irrational solutions

CK ⊕

$\frac{-2+\sqrt{14}}{2}=-1+\frac{\sqrt{14}}{2}$

$\therefore 2\left(-1+\frac{\sqrt{14}}{2}\right)^2+4\left(-1+\frac{\sqrt{14}}{2}\right)-5\stackrel{?}{=}0$

$2\left[1-\frac{\sqrt{14}}{2}-\frac{\sqrt{14}}{2}+\frac{14}{4}\right]-4+\frac{4\sqrt{14}}{2}-5\stackrel{?}{=}0$

$2\left[1-\frac{2\sqrt{14}}{2}+\frac{7}{2}\right]-9+\frac{4\sqrt{14}}{2}\stackrel{?}{=}0$

$2-\frac{4\sqrt{14}}{2}+7-9+\frac{4\sqrt{14}}{2}\stackrel{?}{=}0$ ✓

Ex 4 Pg 315

$$x^2 - 4x = -13$$

$$x^2 - 4x + 13 = 0$$

$a = 1$, $b = -4$, $c = 13$

$$b^2 - 4ac$$

$$(-4)^2 - 4(1)(13)$$

$$16 - 52 = -36 = d$$

$$x = \frac{-b \pm \sqrt{d}}{2a} = \frac{4 \pm \sqrt{-36}}{2(1)}$$

$$x = \frac{4 \pm 6i}{2}$$

$$\boxed{x = 2 \pm 3i}$$

Ck (+)

$$(2+3i)^2 - 4(2+3i) \stackrel{?}{=} -13$$

$$4 + 12i + 9i^2 - 8 - 12i \stackrel{?}{=} -13$$

$$4 - 9 - 8 \stackrel{?}{=} -13 \quad (i^2 = -1) \quad \checkmark$$

Homework: ① Be able to complete the square on $ax^2 + bx + c = 0$ to get the QF. (memorize the QF) too

② Pg 318 # 14, 15, 16, 18 (for conjugates, check ONE of two).