BE - Geometry 11 - Monday 4-18-11

1. Define a function.

2. Define the domain and range of a function.

3. If $y = f(x) = 3x^2$

   $y = g(x) = 2x - 1$

   (A) $f(-3) = ?$

   (b) $g(0) = ?$

   (c) $f[g(x)] = (f \circ g)(x) = ?$

$f(g(x)) \Rightarrow f(x) = 3x^2$

$f(2x-1) = 3(2x-1)^2$

$= 3[4x^2 - 4x + 1]$

$= \boxed{12x^2 - 12x + 3}$

The composition f of g

A function is a set of ordered pairs where each x is paired with one unique y.

(Ex)

x	y
2	6
4	-1
0	6
3	5
9	-4
π	$\sqrt{3}$

Graphs of functions pass the VLT $\Rightarrow$ vertical line test.

Can you name some functions we have covered so far?

OK, for these functions, give their name, domain, and range:

$y = f(x) = ax + b$ — linear function
$d = \mathbb{R}$, $r = \mathbb{R}$

$y = f(x) = ax^2 + bx + c$ — quadratic function
$d = \mathbb{R}$, $r = \mathbb{P}$

$y = f(x) = |x|$ — Absolute value function
$d = \mathbb{R}$, $r = y \geq 0$

$y = f(x) = \sin(x)$ — sine function
$d = \mathbb{R}$, $r = -1 \leq y \leq 1$

$y = f(x) = \arcsin(x)$ ($\sin^{-1}(x)$)
$d = -1 \leq x \leq 1$ $r = \mathbb{R}$
arcsin function or inverse sine function

$y = f(x) = b^x$ — exponential function
$d = \mathbb{P}$ $r = y > 0$

$y = f(x) = \log_b x$ — logarithmic function or inverse exponential funct.
$d = x \geq 0$ $y = \mathbb{R}$

So what is an inverse function?

Inverse Function

Symbol
$f^{-1}(x)$

Two functions are inverse if, for every ordered pair (a, b) in one function, the other function contains (b, a)

In other words, the x and y values are swapped for every (x, y)

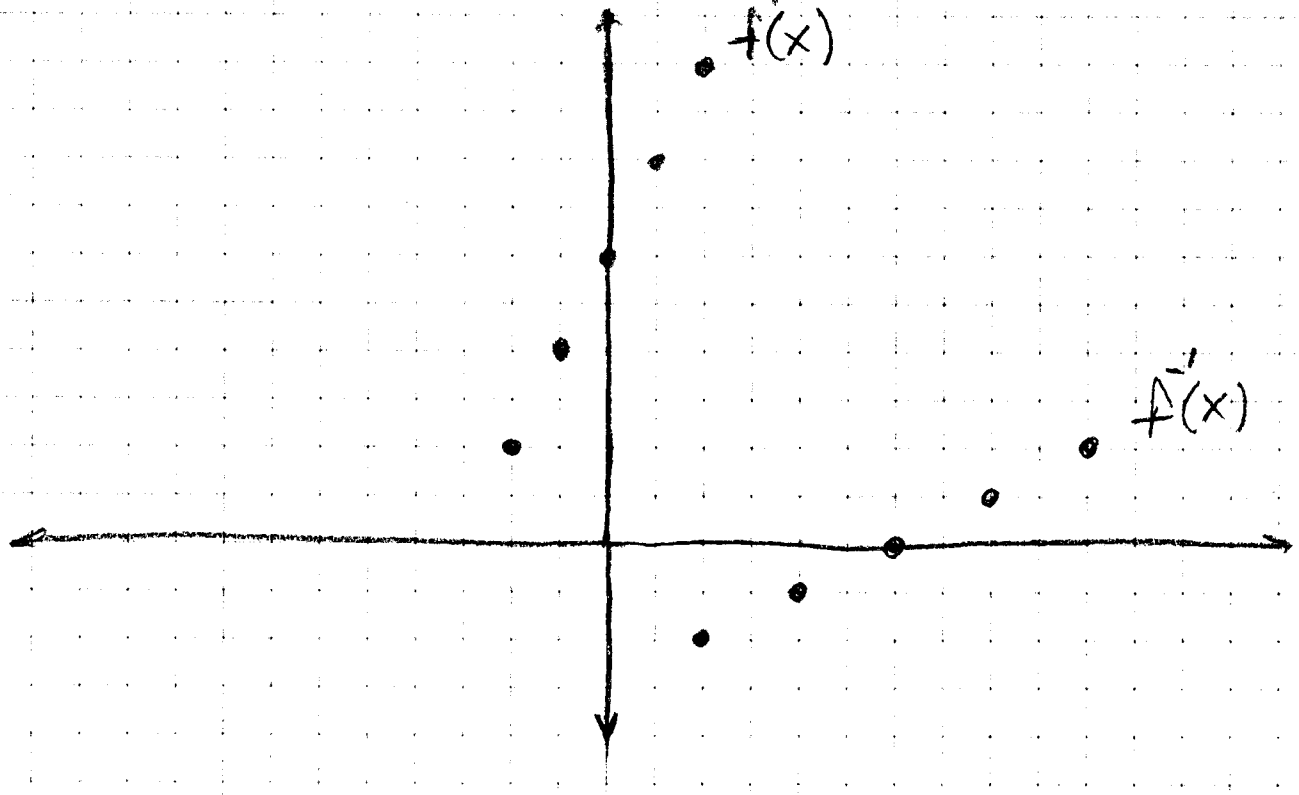
(Ex) $f(x)$

x	y
-2	2
-1	4
0	6
1	8
2	10

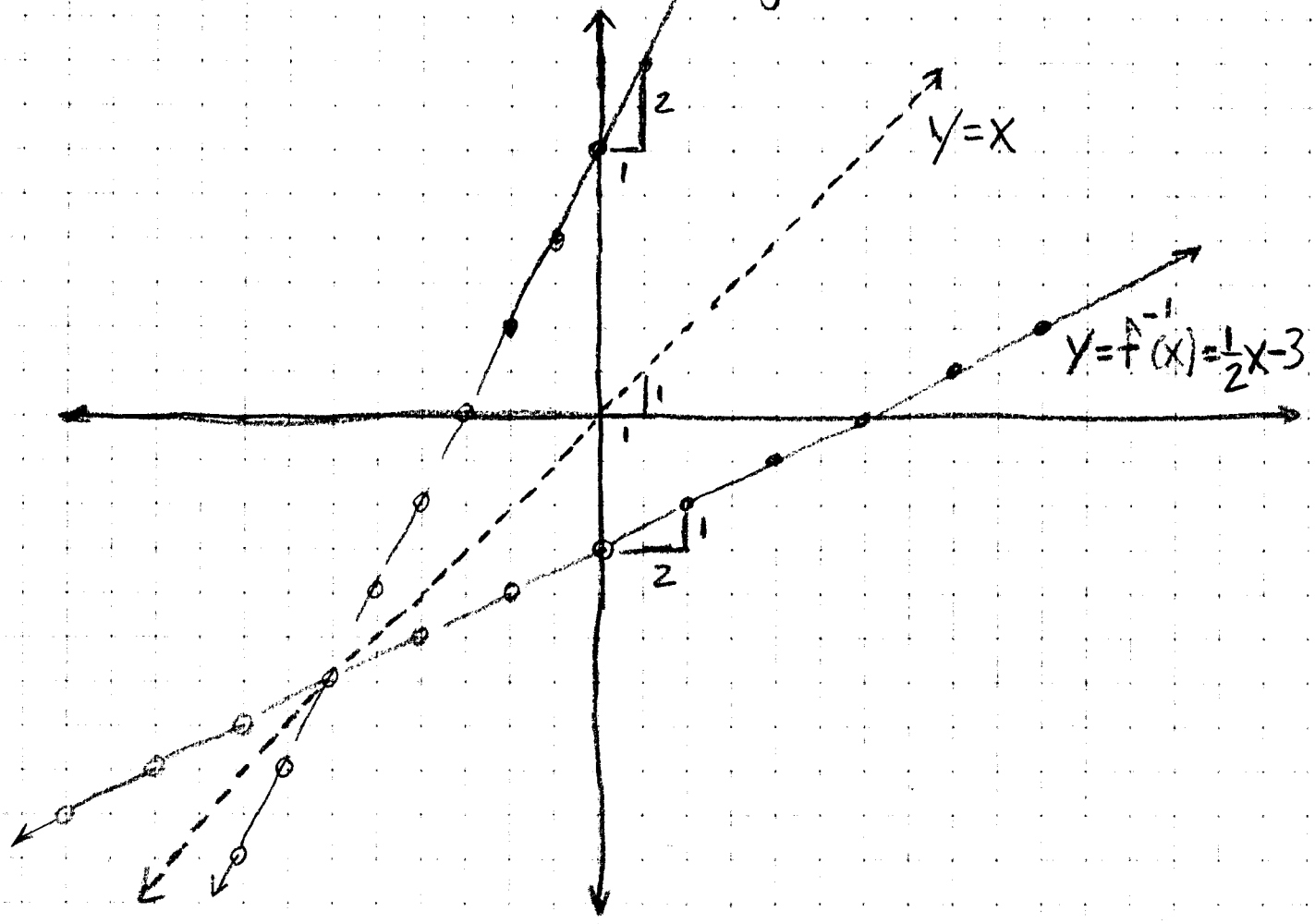
$f^{-1}(x)$

x	y
2	-2
4	-1
6	0
8	1
10	2

Is There A Relationship?



Look at these 3 lines $y = f(x) = 2x + 6$



INVERSE FUNCTIONS ARE ALWAYS SYMMETRICAL ABOUT THE LINE $y = x$

NOT ALL FUNCTIONS HAVE INVERSE FUNCTIONS BUT ... HERE IS HOW TO FIND THEM:

(Ex) $y = f(x) = 2x + 6$

SWAP x AND y AND SOLVE FOR y

$$x = 2y + 6$$

$$\frac{x-6}{2} = \frac{2y}{2}$$

$$\frac{1}{2}x - 3 = y = f^{-1}(x)$$

Find the inverse and graph both functions — show they are symmetrical to a dashed line ---→ $y = x$

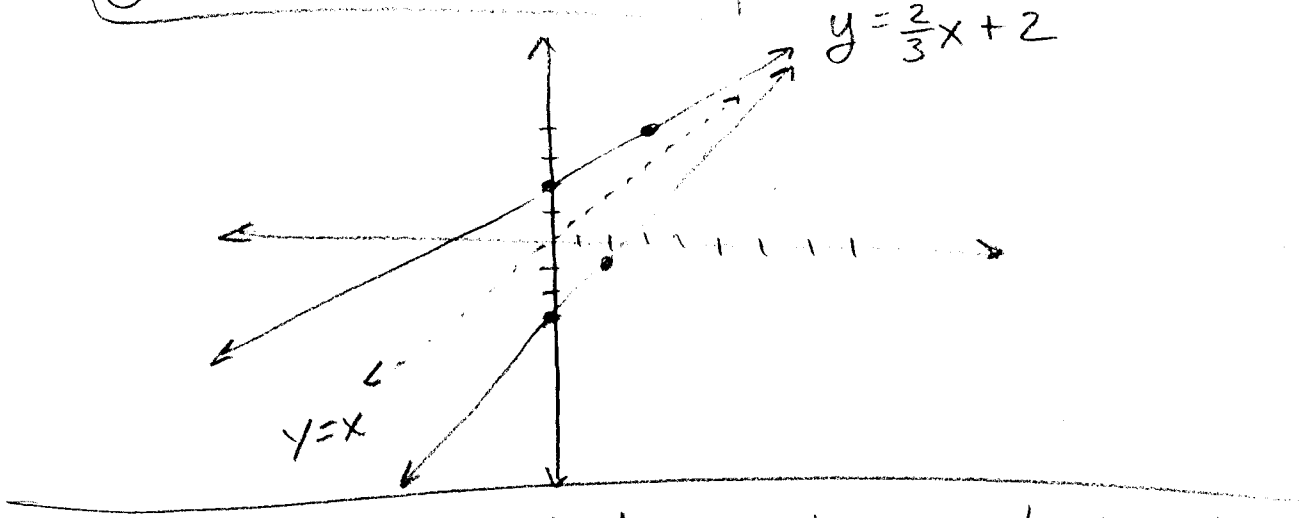
$$y = f(x) = \frac{2}{3}x + 2$$

$$f^{-1}(x) \Rightarrow \quad x = \frac{2}{3}y + 2$$

$$x - 2 = \frac{2}{3}y$$

$$\frac{3}{2}(x - 2) = y$$

$$y = f^{-1}(x) = \frac{3}{2}x - 3$$



Homework: Find $f^{-1}(x)$ and graph both with $y = x$ ←- - - →

① $y = 2x - 4$

② $y = \frac{1}{2}x + 3$

③ $y = 4x - 1$

④ $y = -\frac{3}{4}x + 6$