BE - Geometry I | Monday 8-30-10

1. Solve $3x^2 = 27$

Take $\sqrt{\ }$ of both sides after getting x^2 by itself

2. $\sqrt{16x^2 + 8x + 1} = ?$

PST??? Check it.

Can you put the ideas used to solve 1 & 2 together to solve 3?

3. $25x^2 + 20x + 4 = 144$

PST = Perfect Square Trinomial

$$25x^2 + 20x + 4 = (5x+2)^2$$

$$\therefore \sqrt{25x^2 + 20x + 4} = \sqrt{144}$$

$$\sqrt{(5x+2)^2} = \sqrt{12^2}$$

Look! Must be $\pm\sqrt{12}$ NOT JUST $+\sqrt{12}$ OR JUST $-\sqrt{12}$

$$5x + 2 = \pm 12$$

$$-2 \quad -2$$

$$\frac{5x}{5} = \frac{-2 \pm 12}{5}$$

$$x = \frac{-2 \pm 12}{5} \quad \therefore x = \left\{\frac{-2+12}{5}, \frac{-2-12}{5}\right\}$$

$$\boxed{x = \left\{2, -\frac{14}{5}\right\}}$$

Here is THE TRICK... if the left side of a quadratic trinomial is A Perfect Square Trinomial, you can solve the equation by taking the square root of both sides.

Here is why it is so important...

Any, repeat ANY quadratic can be made into A PST in 3 easy steps — this method is called "Completing The Square" And can be used to solve all quadratics whether they are factorable or Not.

$x = \{\text{rational}\}$ (Ex) $\frac{14}{5}$

$x = \{\text{Not rational}\}$ (Ex) $\sqrt{5}$

BTW:

Since $X = \{2, -\frac{14}{5}\}$ = rational, the original equation must have been factorable so it could have be solved by factoring = "the magic number" method:

$$25x^2 + 20x + 4 = 144$$

Subtract 144 to put in $ax^2+bx+c=0$ form, then GCF.

$$25x^2 + 20x - 140 = 0$$

$$5\,(\underbrace{5x^2 + 4x - 28}) = 0$$

sum $= b = 4$

prod $= ac = -140$

$+14 \quad -10$

$$(5x^2 + 14x) + (^-10x - 28)$$

$$x(5x + 14) + {}^-2(5x + 14)$$

$$\therefore 5\left[(5x+14)(x-2)\right] = 0$$

$$\boxed{X = \left\{-\frac{14}{5}, 2\right\}}$$

✓ Same answer as completing the square.

3 steps to Complete the Square!

EXAMPLE ①

① Create a "hole" by moving all numbers to right side. Follow Golden Rule of Equations.

$$2x^2 + 8x - 60 = -7$$

$$2x^2 + 8x + \{\ \} = 53 + \{\ \}$$

② Make the "a" term +1 by ÷ both sides by a

$$\frac{2x^2}{2} + \frac{8x}{2} + \{\ \} = \frac{53}{2} + \{\ \}$$

$$x^2 + 4x + \{\ \} = \frac{53}{2} + \{\ \}$$

③ "Fill the hole" = Complete the square by adding $\left(\frac{1}{2}b\right)^2$ to both sides

$$\frac{4}{2} = 2 \therefore \text{ADD } 2^2$$

$$x^2 + 4x + \{2^2\} = \frac{53}{2} + \{4\}$$

Finish by $\sqrt{\ }$ both sides and simplify

$$x^2 + 4x + 2^2 = \frac{53}{2} + \frac{8}{2}$$

$$(x+2)^2 = \frac{61}{2}$$

$$\sqrt{(x+2)^2} = \pm\sqrt{\frac{61}{2}}$$

$$x + 2 = \pm\sqrt{\frac{61}{2}}$$

$$\boxed{x = -2 \pm \sqrt{\frac{61}{2}}}$$

irrational ⇒ cannot solve by factoring!!!

Simplifying radicals ⇒ many rules

- No perfect square factors left under radical

Ex $\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4}\sqrt{10}$
$= \boxed{2\sqrt{10}}$

problem

simplified

- No radicals in denominator

problem

Ex $\sqrt{\dfrac{15}{4}} = \dfrac{\sqrt{15}}{\sqrt{4}} = \dfrac{\sqrt{15}}{2}$

Ex $\sqrt{\dfrac{15}{3}} = \dfrac{\sqrt{15}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{45}}{3}$

problem

$= \dfrac{\sqrt{9}\sqrt{5}}{3}$

$= \dfrac{3\sqrt{5}}{3}$

$= \boxed{\sqrt{5}}$

simplified

This was earlier answer

Look at $\sqrt{\dfrac{61}{2}} = \dfrac{\sqrt{61}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{122}}{2}$

problem

$\therefore \; x = -2 \pm \sqrt{\dfrac{61}{2}} = -2 \pm \dfrac{\sqrt{122}}{2} = \dfrac{-4}{2} \pm \dfrac{\sqrt{122}}{2}$

$x = \left\{ \dfrac{-4 \pm \sqrt{122}}{2} \right\} = \left\{ \dfrac{-4 + \sqrt{122}}{2}, \dfrac{-4 - \sqrt{122}}{2} \right\}$

Summary ⇒ Complete the Square

1. Create a hole – Follow GRE
2. Make "a" = 1 by ÷ by a
3. Fill hole by taking half of b and squaring it.

Ex #2

$$9a^2 - 18a = 4$$

$$\frac{9}{9}a^2 - \frac{18}{9}a + \{\ \} = \frac{4}{9} + \{\ \}$$

$$a^2 - 2a + \{1^2\} = \frac{4}{9} + \{1^2\}$$

$$a^2 - 2a + 1 = \frac{13}{9}$$

$$(a-1)^2 = \frac{13}{9}$$

$$a - 1 = \pm\sqrt{\frac{13}{9}} = \pm\frac{\sqrt{13}}{3}$$

$$\boxed{a = 1 \pm \frac{\sqrt{13}}{3} \text{ or } \frac{3 \pm \sqrt{13}}{3}}$$

Checks:

$$9a^2 - 18a = 4 \qquad x = \left\{\frac{3+\sqrt{13}}{3}, \frac{3-\sqrt{13}}{3}\right\}$$

CK $\frac{3+\sqrt{13}}{3}$

$$9\left(\frac{3+\sqrt{13}}{3}\right)^2 - \overset{6}{\cancel{18}}\left(\frac{3+\sqrt{13}}{\cancel{3}}\right) \overset{?}{=} 4$$

$$9\left(\frac{3+\sqrt{13}}{3}\right)\left(\frac{3+\sqrt{13}}{3}\right) - \left(18 + 6\sqrt{13}\right) \overset{?}{=} 4$$

$$\cancel{9}\left[\frac{3^2 + 3\sqrt{13} + 3\sqrt{13} + (\sqrt{13})^2}{\cancel{9}}\right] - 18 - 6\sqrt{13} \overset{?}{=} 4$$

$$9 + \underline{6\sqrt{13}} + 13 - 18 - \underline{6\sqrt{13}} \overset{?}{=} 4$$

$$22 - 18 \overset{?}{=} 4 \checkmark$$

CK $\frac{3-\sqrt{13}}{3}$

$$9\left(\frac{3-\sqrt{13}}{3}\right)^2 - 18\left(\frac{3-\sqrt{13}}{3}\right) \overset{?}{=} 4$$

$$9\left[\frac{9 - 6\sqrt{13} + 13}{9}\right] - \left(18 - 6\sqrt{13}\right) \overset{?}{=} 4$$

$$22 - 6\sqrt{13} - 18 + 6\sqrt{13} \overset{?}{=} 4 \checkmark$$

Your checks should look like this = some mental math but no "magic", usually with conjugates
I will accept ONE check; I would do the $\oplus$

EX 11

From worksheet

$$v^2 + 17v + 26 = 10$$

$$v^2 + 17v + \{\ \} = -16 + \{\ \}$$

a is already 1, take $\frac{17}{2}$ and square it

$$\begin{array}{r} 17 \\ \times 17 \\ \hline 119 \\ 17 \\ \hline 289 \end{array}$$

$$v^2 + 17v + \left(\frac{17}{2}\right)^2 = -16 + \frac{289}{4}$$

$$\left(v + \frac{17}{2}\right)^2 = \frac{-64}{4} + \frac{289}{4} = \frac{225}{4}$$

$$v + \frac{17}{2} = \pm\sqrt{\frac{225}{4}} = \pm\frac{15}{2}$$

NOTE: RATIONAL SOLUTION or RATIONAL "ROOTS", could have factored $\Rightarrow$

$$v = -\frac{17}{2} \pm \frac{15}{2} = \frac{-17 \pm 15}{2}$$

$$v = \left\{\frac{-2}{2}, \frac{-32}{2}\right\}$$

$$\boxed{v = \{-1, -16\}}$$

CK $v=-1$

$$(-1)^2 + 17(-1) + 26 \stackrel{?}{=} 10$$

$$1 - 17 + 26 \stackrel{?}{=} 10 \checkmark$$

CK $v=-16$

$$(-16)^2 + 17(-16) + 26 \stackrel{?}{=} 10$$

$$256 - 272 + 26 \stackrel{?}{=} 10 \checkmark$$

HW: DO 3–6 ON WORKSHEET w/CHECKS 1, 2, 11 done ALREADY

Geometry 1 - Mr. C. - All work on looseleaf. Name________________________________

HW Practice - Completing the Square

Date________________ Period____

Solve each equation by completing the square.

1) $2m^2 + 8m - 60 = -7$

(SEE CLASS NOTES)

2) $9a^2 - 18a = 4$

(SEE CLASS NOTES)

3) $6p^2 + 12p - 41 = 7$

4) $9x^2 + 18x + 14 = 7$

5) $8x^2 + 16x + 16 = 10$

6) $2n^2 - 4n - 82 = 2$

7) $8r^2 - 16r + 8 = 2$

8) $10x^2 - 20x - 20 = 10$

9) $n^2 - 20n + 95 = -4$

10) $b^2 + 14b - 80 = -2$

11) $v^2 + 17v + 26 = 10$

(SEE CLASS NOTES)

12) $x^2 - 7x - 52 = -8$