

# ALGEBRA I

The Algebra I course builds on foundational mathematics content learned by students in Grades K-8 by expanding mathematics understanding to provide students with a strong mathematics education. Content is designed to engage students in a variety of mathematical experiences that include the use of reasoning and problem-solving skills, which may be applied to life situations beyond the classroom setting. This course serves as the cornerstone for all high school mathematics courses; therefore, all subsequent mathematics courses require student mastery of the Algebra I content standards.

Algebra I is one of the courses required for all students. School systems may offer Algebra I and Algebra IA and Algebra IB. Content standards 3, 4, 5, 6, 7, 7a, 15, 16, 21, 22, 24, 25, 26, 29, 30, 30a, 37, 37b, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, and 50 must be taught in the Algebra IA course. Content standards 1, 2, 7b, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 23, 27, 28, 30b, 30c, 31, 32, 33, 34, 35, 36, 37a, 37c, and 38 must be taught in the Algebra IB course. Systems offering Algebra I in the eighth grade have the responsibility of ensuring that all Algebra I course content standards and Grade 8 course content standards be included in instruction.

Students will:

## NUMBER AND QUANTITY

### The Real Number System

#### Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. [N-RN1]  
Example: We define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents. [N-RN2]

#### Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. [N-RN3]

## Quantities\*

**Reason quantitatively and use units to solve problems.** (*Foundation for work with expressions, equations, and functions.*)

4. Use units as a way to understand problems and to guide the solution of multistep problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. [N-Q1]
5. Define appropriate quantities for the purpose of descriptive modeling. [N-Q2]
6. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. [N-Q3]

## ALGEBRA

### Seeing Structure in Expressions

**Interpret the structure of expressions.** (*Linear, exponential, quadratic.*)

7. Interpret expressions that represent a quantity in terms of its context.\* [A-SSE1]
  - a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
 

Example: Interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .
8. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
 

Example: See  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .

**Write expressions in equivalent forms to solve problems.** (*Quadratic and exponential.*)

9. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\* [A-SSE3]
  - a. Factor a quadratic expression to reveal the zeros of the function it defines. [A-SSE3a]
  - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. [A-SSE3b]
  - c. Determine a quadratic equation when given its graph or roots. 
  - d. Use the properties of exponents to transform expressions for exponential functions. [A-SSE3c]
 

Example: The expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

### Arithmetic With Polynomials and Rational Expressions

**Perform arithmetic operations on polynomials.** (*Linear and quadratic.*)

10. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

**Creating Equations\***

**Create equations that describe numbers or relationships.** (*Linear, quadratic, and exponential (integer inputs only); for Standard 13, linear only.*)

11. Create equations and inequalities in one variable, and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]
12. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]
13. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. [A-CED3]  
 Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.
14. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]  
 Example: Rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

**Reasoning With Equations and Inequalities**

**Understand solving equations as a process of reasoning and explain the reasoning.** (*Master linear; learn as general principle.*)

15. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. [A-REI1]

**Solve equations and inequalities in one variable.** (*Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions.*)

16. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. [A-REI3]
17. Solve quadratic equations in one variable. [A-REI4]
  - a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form. [A-REI4a]
  - b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square and the quadratic formula, and factoring as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions, and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ . [A-REI4b]

## Solve systems of equations. (*Linear-linear and linear-quadratic.*)

18. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. [A-REI5]
19. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. [A-REI6]
20. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. [A-REI7]  
 Example: Find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .

## Represent and solve equations and inequalities graphically. (*Linear and exponential; learn as general principle.*)

21. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [A-REI10]
22. Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\* [A-REI11]
23. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. [A-REI12]

## FUNCTIONS

### Interpreting Functions

#### Understand the concept of a function and use function notation. (*Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences.*)

24. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . [F-IF1]
25. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. [F-IF2]
26. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. [F-IF3]  
 Example: The Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .

**Interpret functions that arise in applications in terms of the context. (*Linear, exponential, and quadratic.*)**

27. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\** [F-IF4]
28. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.\* [F-IF5]
 

Example: If the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.
29. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\* [F-IF6]

**Analyze functions using different representations. (*Linear, exponential, quadratic, absolute value, step, piecewise-defined.*)**

30. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\* [F-IF7]
  - a. Graph linear and quadratic functions, and show intercepts, maxima, and minima. [F-IF7a]
  - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
  - c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]
31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]
  - a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. [F-IF8a]
  - b. Use the properties of exponents to interpret expressions for exponential functions. [F-IF8b]
 

Example: Identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ , and  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth and decay.
32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
 

Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

## Building Functions

**Build a function that models a relationship between two quantities.** (*For standards 33 and 34, linear, exponential, and quadratic.*)

33. Write a function that describes a relationship between two quantities.\* [F-BF1]
- Determine an explicit expression, a recursive process, or steps for calculation from a context. [F-BF1a]
  - Combine standard function types using arithmetic operations. [F-BF1b]  
 Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
34. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\* [F-BF2]

**Build new functions from existing functions.** (*Linear, exponential, quadratic, and absolute value; for standard 36a, linear only.*)

35. Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]
36. Find inverse functions. [F-BF4]
- Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse, and write an expression for the inverse. [F-BF4a]  
 Example:  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .

## Linear, Quadratic, and Exponential Models\*

**Construct and compare linear, quadratic, and exponential models and solve problems.**

37. Distinguish between situations that can be modeled with linear functions and with exponential functions. [F-LE1]
- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. [F-LE1a]
  - Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. [F-LE1b]
  - Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. [F-LE1c]
38. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). [F-LE2]
39. Observe, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. [F-LE3]

**Interpret expressions for functions in terms of the situation they model. (*Linear and exponential of form  $f(x) = b^x + k.$* )**

40. Interpret the parameters in a linear or exponential function in terms of a context. [F-LE5]

## **STATISTICS AND PROBABILITY**

### **Interpreting Categorical and Quantitative Data**

**Summarize, represent, and interpret data on a single count or measurement variable.**

41. Represent data with plots on the real number line (dot plots, histograms, and box plots). [S-ID1]
42. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. [S-ID2]
43. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). [S-ID3]

**Summarize, represent, and interpret data on two categorical and quantitative variables. (*Linear focus, discuss general principle.*)**

44. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. [S-ID5]
45. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. [S-ID6]
- Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* [S-ID6a]
  - Informally assess the fit of a function by plotting and analyzing residuals. [S-ID6b]
  - Fit a linear function for a scatter plot that suggests a linear association. [S-ID6c]

**Interpret linear models.**

46. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. [S-ID7]
47. Compute (using technology) and interpret the correlation coefficient of a linear fit. [S-ID8]
48. Distinguish between correlation and causation. [S-ID9]

**Conditional Probability and the Rules of Probability**

**Understand independence and conditional probability and use them to interpret data.**

*(Link to data from simulations or experiments.)*

49. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). [S-CP1]
50. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. [S-CP2]