

Ch. 8-8 Completing the Square

Concept: if your quadratic is factorable and a perfect square trinomial, it is easy to solve because you can just take the square root of both sides

(EX) $x^2 + 6x + 9 = 4$
 ↓ ↓
 ↓ ↓
 $(x + 3)^2 = 4$

$$\sqrt{(x+3)^2} = \sqrt{4}$$

$$x + 3 = \pm 2$$

$$x = -3 \pm 2$$

$$x = \{-3 + 2, -3 - 2\}$$

$$x = \{-1, -5\}$$

Here is the good news - you can turn any quadratic into a perfect square trinomial !!!

This process is known as "Completing the square"

- Steps:
- ① Put into standard form for quadratic equation.
 - ② Create placeholder for new 3rd term by moving the "c" term to right side.
 - ③ If a is not +1, divide all terms by a to make it +1.
 - ④ Complete the square by taking $\frac{b}{2}$ and putting $(\frac{b}{2})^2$ as 3rd term, * don't forget GRE
 - ⑤ take PST; square root both sides
 - ⑥ Do checks!

$$\textcircled{\text{EX}} \quad 3x^2 - 10x = -3$$

$$3x^2 - 10x + 3 = 0$$

SF, NO GCF

$$3x^2 - 10x + \{\} = -3 + \{\}$$

"Double your bubble, to stay out of trouble"

$$\uparrow \\ a \neq 1 \therefore \div \text{ by } 3$$

$$\frac{3x^2}{3} - \frac{10x}{3} + \{\} = \frac{-3}{3} + \{\}$$

$$x^2 - \frac{10}{3}x + \left\{\frac{5}{3}\right\}^2 = -1 + \left\{\frac{25}{9}\right\}$$

$$\uparrow \\ \frac{10}{3} \cdot \frac{1}{2} = \frac{10}{6} = \frac{5}{3}$$

NOTE

$$\frac{-9}{9} + \frac{25}{9} = \frac{16}{9}$$

THIS "completes the square"
you now have a PST on left

$$\left(x - \frac{5}{3}\right)^2 = \frac{16}{9}$$

$$\sqrt{\left(x - \frac{5}{3}\right)^2} = \pm \sqrt{\frac{16}{9}} = \pm \frac{4}{3}$$

↑
DON'T FORGET ±

$$x - \frac{5}{3} = \pm \frac{4}{3}$$

$$x = \frac{5}{3} \pm \frac{4}{3}$$

$$x = \left\{\frac{9}{3}, \frac{1}{3}\right\} = \left\{3, \frac{1}{3}\right\}$$

EX SAME AS ONE ON prior page but done how I would really show it:

$$3x^2 - 10x = -3$$

$$\frac{3}{3}x^2 - \frac{10}{3}x + \left(\frac{5}{3}\right)^2 = \frac{-3}{3} + \left(\frac{25}{9}\right)$$

$$\left(x - \frac{5}{3}\right)^2 = \frac{16}{9}$$

$$x - \frac{5}{3} = \pm \frac{4}{3}$$

$$x = \frac{5}{3} \pm \frac{4}{3} = \left\{ 3, \frac{1}{3} \right\}$$

BTW, THE ORIGINAL QUADRATIC WAS factorable. CTS WORKS whether it is or not.

$$\textcircled{\text{EX}} \quad x^2 - 2x - 2 = 0$$

$$x^2 - 2x + \{1\} = 2 + \{1\}$$

$$\frac{b}{2} = \frac{2}{2} = 1 \quad \swarrow$$

$$\downarrow \quad \downarrow$$

$$(x-1)^2 = 3$$

$$x-1 = \pm \sqrt{3}$$

$$x = 1 \pm \sqrt{3}$$

$$\therefore \boxed{x = \{1 + \sqrt{3}, 1 - \sqrt{3}\}} \quad \text{EXACT}$$

$$\therefore \quad \approx 2.7321 \quad \approx 0.7321 \quad \text{APPROX}$$

NOTES: use EXACT ANSWERS FOR CHECKS, IF YOU HAVE IRRATIONAL CONJUGATES, FOR THIS CLASS, YOU ONLY NEED TO CHECK ONE. TIP: USE + ONE

if you get a \ominus under the $\sqrt{\ominus}$, STOP, ANSWER IS **No Real Solution**

CK $(1 + \sqrt{3})^2 - 2(1 + \sqrt{3}) - 2 \stackrel{?}{=} 0$

$x + \sqrt{3}$ $(1 + \sqrt{3})(1 + \sqrt{3}) - 2 - 2\sqrt{3} - 2 \stackrel{?}{=} 0$

$\underline{1^2} + 1\sqrt{3} + 1\sqrt{3} + \underline{3} - 4 - 2\sqrt{3} \stackrel{?}{=} 0$

$\downarrow \quad \downarrow$

$2\sqrt{3} + 4 - 4 - 2\sqrt{3} \stackrel{?}{=} 0 \quad \checkmark$

$$\textcircled{3} \quad -58 - 16x = -2x^2$$

$$\frac{2x^2}{2} - \frac{16x}{2} - \frac{58}{2} = 0$$

$$x^2 - 8x + \boxed{4^2} = 29. \quad + \boxed{16}$$

$$\downarrow \quad \downarrow \\ (x - 4)^2 = 45$$

$$x - 4 = \pm \sqrt{45} = \pm 3\sqrt{5}$$

$$\boxed{x = 4 \pm 3\sqrt{5}}$$

$$\underline{\underline{or}} \quad x = \{4 + 3\sqrt{5}, 4 - 3\sqrt{5}\}$$

CK
(4 + 3√5)

$$-58 - 16(4 + 3\sqrt{5}) \stackrel{?}{=} -2(4 + 3\sqrt{5})(4 + 3\sqrt{5})$$

$$-58 - 64 - 48\sqrt{5} \stackrel{?}{=} -2[16 + 24\sqrt{5} + 45]$$

$$-122 - 48\sqrt{5} \stackrel{?}{=} -122 - 48\sqrt{5} \checkmark$$

$$\textcircled{6} \quad -6N = -5 - N^2$$

$$N^2 - 6N + \{3^2\} = -5 + \{9\}$$

$$(N - 3)^2 = 4$$

$$N - 3 = \pm 2$$

$$N = 3 \pm 2$$

$$N = \{5, 1\}$$

CK
u=5 ✓