Math 112 Thurs. 11-8-12 CLASS NOTES

11 Find possible rational zeros

Pg 453

$$P(x) = 4x^4 + x^3 + x^2 + x - 3$$

$\pm \frac{p}{q}$ $\Rightarrow$ $4 \Rightarrow 1, 2, 4$

$3 \Rightarrow 1, 3$

Possible rational zeros

$\frac{1}{1}, \frac{3}{1}$

$\frac{1}{2}, \frac{3}{2}$

$\frac{1}{4}, \frac{3}{4}$

$$\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm 3$$

4 TOTAL

Descartes Ros (Rule of Signs)

$P(x) \Rightarrow + + + + - \Rightarrow$ 1 change $\Rightarrow$ 1 positive real zero

$P(-x) \Rightarrow + - + - - \Rightarrow$ 3 changes $\Rightarrow$ 3 or 1 Neg. real zeros

TOTAL	Real (+)	Real (−)	Complex
4	1	3	0
4	1	1	2

Sign TABLE

Ck $x = 1$ Using Synthetic Division

1	4	1	1	1	−3
		4	5	6	7
	4	5	6	7	+4

NOT A zero

−1	4	1	1	1	−3
		−4	3	−4	+3
	4	−3	4	−3	0

$x = -1$ is A zero

$\frac{1}{2}$	4	1	1	1	−3
		2	$\frac{3}{2}$	$\frac{5}{4}$	$\frac{9}{8}$
	4	3	$\frac{5}{2}$	$\frac{9}{4}$	

NOT A zero

$-\frac{1}{2}$	4	1	1	1	−3
		−2	$\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{1}{8}$
	4	−1	$\frac{3}{2}$	$\frac{1}{4}$	

NOT A zero

$\frac{3}{4}$ | 4 1 1 1 −3

* 3 3 3 3 *

4 4 4 4 0

$x = \frac{3}{4}$ is a positive zero

Descartes Rule of Signs ⇒ DONE CHECKING POSITIVE ZEROS

$-\frac{3}{4}$ | 4 1 1 1 −3

$-3 + \frac{3}{2} - \frac{15}{8} + \frac{21}{32}$

4 -2 $\frac{5}{2}$ $-\frac{7}{8}$ NOT A ZERO

$-\frac{1}{4}$ | 4 1 1 1 −3

-1 0 $-\frac{1}{4}$ $-\frac{3}{16}$

4 0 1 $\frac{3}{4}$ NOT A ZERO

$-\frac{3}{2}$ | 4 1 1 1 −3

$-6 + \frac{15}{2} - \frac{51}{4} + \frac{141}{8}$

4 -5 $\frac{17}{2}$ $-\frac{47}{4}$ NOT A ZERO

* Since -3 is only remaining possible negative zero, it cannot work since there must be 3 or 1 negative zeros and we already have $x = -1$, so last 2 must be complex.

Using the 2 real zeros we found by synthetic division, $(x+1)(x-\frac{3}{4})$ must be factors: NOTE $x-\frac{3}{4}=0$ means $(4x-3)=0$

$\therefore (x+1)(4x-3)$ must be a factor

$(4x^2+x-3)$ must be a factor

$\therefore$ $4x^2+x-3$ must go in evenly

$$\begin{array}{r} x^2+0+1 \\ \hline 4x^2+x-3 \;\big|\; 4x^4+x^3+x^2+x-3 \\ \ominus\ \ominus\ \oplus \\ (4x^4+x^3-3x^2) \\ \hline 4x^2+x-3 \\ 4x^2+x-3 \\ \hline 0 \end{array}$$ ✓

$\therefore x^2+1$ is remaining factor

$x^2+1=0$ by ZPP

$x^2=-1$

$x=\pm i$ LAST 2 ZEROS

$\therefore x=\{-1, \frac{3}{4}, +i, -i\}$

WolframAlpha computational knowledge engine

Solve 0=4x^4+x^3+x^2+x-3

Examples Random

Input interpretation:

solve	$0 = 4x^4 + x^3 + x^2 + x - 3$

Results:

Step-by-step solution

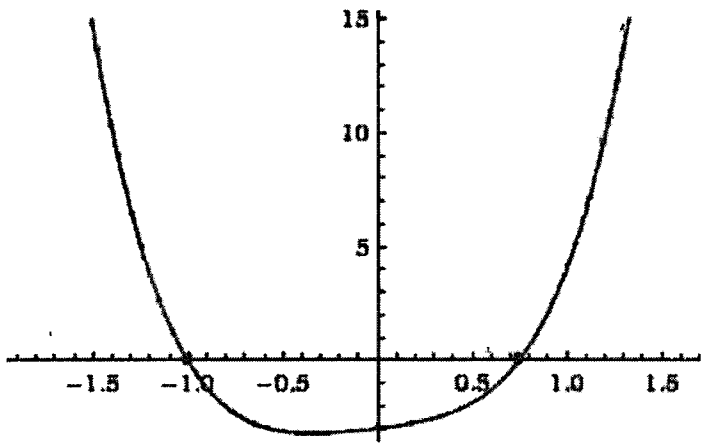
$x = -1$

$x = \pm i$

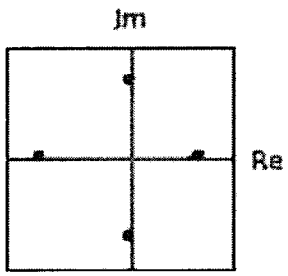
$x = \frac{3}{4}$

NOTE "STEP BY STEP" SOLUTION IS AVAILABLE BY CLICKING here. It doesn't tell you how to come up with the 3 factors ☹

Root plot:



Roots in the complex plane:



NOTE THE COMPLEX PLANE GRAPH OF THE ZEROS,

X AXIS ⇒ "pure" real zeros, $-1, \frac{3}{4}$

Y AXIS ⇒ "pure" imaginary zeros, $\pm i$

(23) Pg 445 Find all zeros of each poly. when one zero is given.

$P(x) = x^3 + x^2 - 20x - 50$; $-3-i$, $-3+i$

$(x - [-3-i])(x - [-3+i])$

$((x+3)+i)((x+3)-i)$

$a + b \quad a - b$

$x^2 + 6x + 9 - (-1)$

$(x^2 + 6x + 10)$

$$x^2+6x+10 \;\overline{)\; x^3 + x^2 - 20x - 50}$$

$x - 5$

$-(x^3 + 6x^2 + 10x)$

$-5x^2 - 30x - 50$

$-5x^2 - 30x - 50$

$\therefore \boxed{x = \{-3-i, -3+i, 5\}}$

$$M = \frac{2\log\left(\frac{E}{10^{4.4}}\right)}{3}$$

Earthquake Problems

Check

$$5.1 \stackrel{?}{=} \frac{2\log\left(\frac{1.1\times10^{12}}{10^{4.4}}\right)}{3}$$

$$= \frac{2(7.6413\ldots)}{3}$$

$$5.1 = 5.094 \checkmark$$

$$M = \frac{2\log\left(\frac{7.08\times10^{16}}{10^{4.4}}\right)}{3}$$

$$8.3 = \frac{2(12.45\ldots)}{3}$$

$$\boxed{M = 8.3}$$

$$A = A_0(1 \pm r)^x$$

↑ Principal

Compound Interest Problems

$$A = P(1+r)^t \qquad r \Rightarrow \text{per year}$$

$$A = P\left(1+\frac{r}{N}\right)^{Nt} \quad \text{CHUNKY}$$

$$A = Pe^{rt} \quad \text{CONTINUOUS}$$

↑ BASE e

(4) (EX) (A)

$P = \$5000$

$t = 2$ years

$r = 6.5\% = .065$

$N = 4$

$$A = 5000\left(1+\frac{.065}{4}\right)^{4(2)}$$

$$A = 5000(1.01625)^8$$

$$= 5000(1.137638)$$

$$= 5688.19495$$

$$A = \$5688.19$$

(B)

$P = 5000$

$t = 2$ years

$r = 5.5\% = .055$

$N = 52$

$$A = 5000\left(1+\frac{.055}{52}\right)^{52\cdot 2}$$

$$A = 5000(1.001057692)^{104}$$

$$A = 5000(1.116213\ldots)$$

$$A = 5581.0659$$

$$A = \$5581.07$$