

Mth113

Fri. 1-11-13 (Class Notes)

Trig. Identities (cont).

Tip $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$

$\cos^2 \theta = 1 - \sin^2 \theta$

these can be factored leading to linear expressions!

$$1 - \cos^2 \theta = ?$$

$$\frac{(1 + \cos \theta)(1 - \cos \theta)}{}$$

$$1 - \sin^2 \theta = ?$$

$$(1 + \sin \theta)(1 - \sin \theta)$$

EOT
 (41)
 PS152

$$\cos^4 X - \sin^4 X = \cos^2 X - \sin^2 X$$

$$(\cos^2 X - \sin^2 X)(\cos^2 X + \sin^2 X)$$

$$\cos^2 X - \sin^2 X$$

$$\cos^2 X - \sin^2 X$$

$$\textcircled{40} \frac{(1 + \tan^2 \theta)}{\tan^2 \theta}$$

$$= \csc^2 \theta$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\textcircled{\tan^2 \theta + 1 = \sec^2 \theta}$$

$$\frac{\sec^2 \theta}{\tan^2 \theta}$$

$$\frac{\frac{1}{\cos^2 \theta}}{\frac{\sin \theta}{\cos \theta}} = \frac{1}{\sin^2 \theta}$$

$$\downarrow$$

$$\csc^2 \theta$$

$$\downarrow$$

$$\csc^2 \theta \quad \checkmark$$

$$\textcircled{42} \quad \frac{1 + \sec x}{\sec x} = \frac{\sin^2 x}{1 - \cos x} \quad \left\{ \begin{array}{l} \text{Hint} \\ \sin^2 x = \\ (1 - \cos x)(1 + \cos x) \end{array} \right.$$

$$\frac{1}{\sec x} + \frac{\sec x}{\sec x}$$

↓

$$\cos x + 1$$

$$\frac{\cancel{(1 - \cos x)}(1 + \cos x)}{\cancel{(1 - \cos x)}}$$

$$1 + \cos x \quad \checkmark$$

$$\textcircled{49} \quad \sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$$

$$\frac{\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}}{\frac{\sin^2 x}{\cos^2 x \sin^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x}}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$$

$$\frac{1}{\cos^2 x \sin^2 x}$$

$$\frac{1}{\cos^2 x} \left(\frac{1}{\sin^2 x} \right)$$

↓

$$\frac{1}{\cos^2 x \sin^2 x} \quad \checkmark$$

1) Hint:

$$(43) \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Since $(1 - \cos x)(1 + \cos x) = 1 - \cos^2 x$

$$\frac{1 - \cos^2 x}{(1 + \cos x) \sin x}$$

$$\frac{\sin^2 x}{(1 + \cos x) \sin x}$$

$$\frac{\sin x}{1 + \cos x}$$

$$\frac{\sin x}{1 + \cos x}$$

$$\frac{\sin x}{1 + \cos x} \quad \checkmark$$

Hint: $\frac{\sin x}{1 + \cos x} = \frac{\sin x (1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$

ANOTHER WAY TO SOLVE

TO SOLVE

$$\textcircled{50} \quad \tan^4 x + \tan^2 x = \sec^4 x - \sec^2 x$$

$$\tan^2 x (\tan^2 x + 1)$$

$$\Downarrow$$
$$\tan^2 x + 1 = \sec^2 x$$

$$\tan^2 x (\sec^2 x)$$

$$\Downarrow$$
$$\sec^2 x - 1 (\sec^2 x)$$

$$\sec^4 x - \sec^2 x$$

$$\sec^4 x - \sec^2 x$$