Mth 113 TUESDAY 2-12-13 CLASS NOTES

Ch. 5-3 The Double-Angle and Half-Angle Identities

Double-Angle Identities

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
  $= 1 - 2\sin^2\alpha$
  $= 2\cos^2\alpha - 1$
- $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

PUT ON ONE SIDE OF 3x5 CARD.

Half-Angle Identities

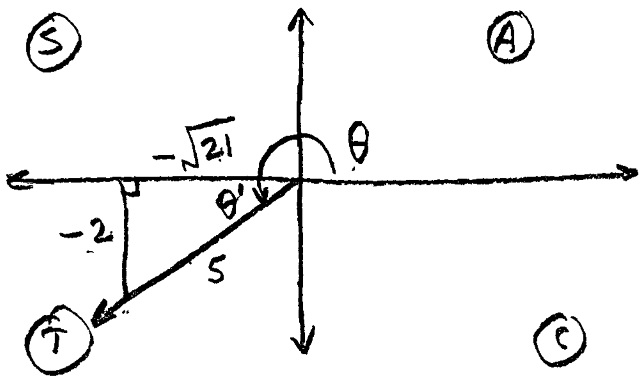
Note: + or − from ASTC

- $\sin\frac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$
- $\cos\frac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$
- $\tan\frac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$
  $= \dfrac{\sin\alpha}{1+\cos\alpha}$
  $= \dfrac{1-\cos\alpha}{\sin\alpha}$

PUT ON ONE SIDE OF 3x5 CARD

(EX 1) If $\sin\theta = -\frac{2}{5}$, θ in Quadrant III

Pg 184 Find Exact values of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$



$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(-\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right) = \boxed{+\frac{4\sqrt{21}}{25}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(-\frac{\sqrt{21}}{5}\right)^2 - \left(-\frac{2}{5}\right)^2$$

$$= \frac{21}{25} - \frac{4}{25} = \boxed{\frac{17}{25}}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{2}{\sqrt{21}}\right)}{1-\left(\frac{2}{\sqrt{21}}\right)^2} = \frac{\frac{4}{\sqrt{21}}}{\frac{21}{21}-\frac{4}{21}}$$

$$= \frac{\frac{4}{\sqrt{21}}}{\frac{17}{21}} = \frac{4\cdot 21}{17\sqrt{21}} = \frac{84}{17\sqrt{21}} = \frac{\cancel{84}^{4}\sqrt{21}}{17\cdot\cancel{21}_{1}}$$

$$= \boxed{\frac{4\sqrt{21}}{17}}$$

Using Double-Angle and Half-Angle Identities to find "new" exact values of trig. functions.

EX1 Find the exact value of $\sin 22.5^\circ$

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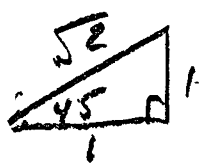
Using $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\Rightarrow$ $$\sin\left(\frac{45}{2}\right) = \pm\sqrt{\frac{1-\cos 45}{2}}$$



$\frac{45}{2}$ is in Quadrant I $\therefore \sin\frac{45}{2}$ is $+$

$$\sin\left(\frac{45}{2}\right) = +\sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

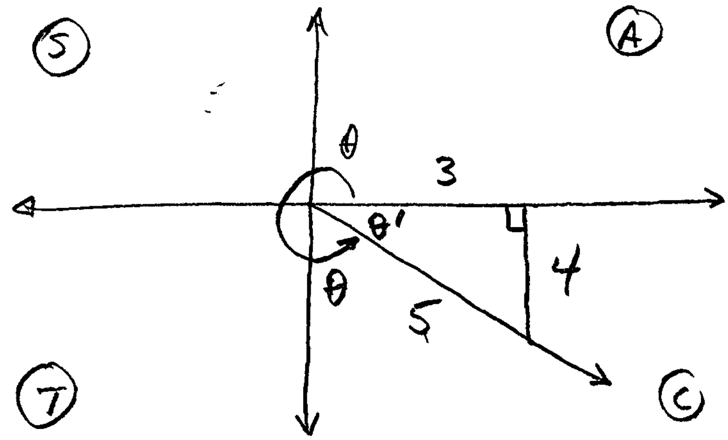
$$\boxed{\sin(22.5^\circ) = \frac{\sqrt{2-\sqrt{2}}}{\sqrt{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}}$$

$\approx .3827$

Ex2 pg 187 $\quad \cos\theta = \frac{3}{5}$; $\frac{3\pi}{2} < \theta < 2\pi$

Quadrant IV

Find exact value of $\cos\left(\frac{\theta}{2}\right)$



TIP

Figure out the sign (using ASTC) then the math!!

$$\boxed{\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}}$$

If θ is between $\frac{3\pi}{2}$ and $2\pi = QIV$

$\frac{\theta}{2}$ is between $\frac{3\pi}{4}$ and $\pi = QII$

$\therefore \cos\frac{\theta}{2}$ is $-$

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\frac{3}{5}}{2}}$$

$$= -\sqrt{\frac{\frac{8}{5}}{2}} = -\sqrt{\frac{8}{10}} = -\sqrt{\frac{4}{5}}$$

$$\therefore \boxed{\cos\left(\frac{\theta}{2}\right) = \frac{-\sqrt{4}}{\sqrt{5}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{-2\sqrt{5}}{5}}}$$

Ans

The half and double angle identities can be written in many equivalent forms – this is one of the reasons they are very 'flexible', ie, they can be rewritten into terms more convenient for whatever you are trying to do:

(Ex) — Bottom of Page 187

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$ Double Angle Identity

$$\Rightarrow \sin 4\alpha = 2\sin 2\alpha\cos 2\alpha$$ Another form

$$\Rightarrow \sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$ Another form

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{Ex 1 Pg 188} Rewrite in $a\sin x$, $a\cos x$ or $a\tan x$ form

$$2\sin 4\theta\cos 4\theta$$

Since $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$\Rightarrow 2\sin 4\theta\cos 4\theta = \boxed{\sin 8\theta}$$

Because $\sin 2(4\theta) = 2\sin(4\theta)\cos(4\theta)$

$\sin 2(\alpha) = 2\sin(\alpha)\cos(\alpha)$

Rewrite $\dfrac{4\tan 2\theta}{1-\tan^2 2\theta}$ as $a\tan\theta$

EX2

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$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

if $2\theta = \alpha$

then $\dfrac{2 \cdot 2\tan 2\theta}{1-\tan^2 2\theta} = 2\tan(2)(2\theta)$

$$2\left(\frac{2\tan 2\theta}{1-\tan^2 2\theta}\right) = \boxed{2\tan 4\theta}$$

EX3

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Rewrite $\cos^2(80°) - \sin^2(80°)$ as $a\cos\theta$

$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

$$\cos^2(80) - \sin^2(80) = \boxed{\cos 160°}$$
