Mth 113 Wednesday 2-20-13 Class Notes
(week #7)

Ch 5-4 Conditional Trigonometric Equations

NOTE: Whenever you compute an inverse trig. function to solve an equation, use the absolute value of the argument which gives the reference angle of the answer.
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Primary Solutions solutions to trig. equations that are between $0 \le x < 360^0$ or $0 \le x < 2\pi$

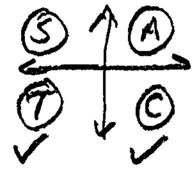
Examples – Pg 193 - 194 # 5-4A

Find all primary solutions, degrees AND radians

degrees ⇒ nearest .1   radians ⇒ nearest .0001

① $5 \sin \alpha = -2$

$\sin \alpha = -\frac{2}{5}$ ∴



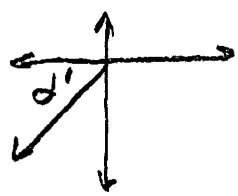
α is in Quadrant III and IV

$\sin^{-1}\left(\left|-\frac{2}{5}\right|\right) = \alpha'$

reference angle

$\alpha' = 23.578° = \alpha'$

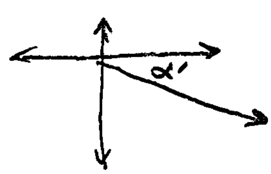
∴ Quad III



$\alpha = 180 + 23.578°$

$\boxed{\alpha = 203.6°}$

Quad IV



$\alpha = 360 - 23.578$

$\boxed{\alpha = 336.4°}$

$\boxed{\alpha = \{203.6°, 336.4°\}}$

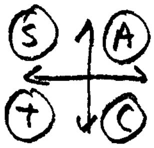
RADIAN MODE $\sin^{-1}\left(\left|-\frac{2}{5}\right|\right) = .4115 = \alpha'$

∴ Quad III $\alpha = \pi + .4115 = \boxed{3.5531 \text{ rad.}}$

Quad IV $\alpha = 2\pi - .4115 = \boxed{5.8717 \text{ rad}}$

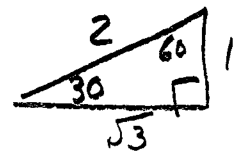
$\boxed{\alpha = \{3.5531, 5.8717\}}$

(2) $\cos\theta = -\frac{1}{2}$ $\therefore\ \theta' = \cos^{-1}\left(\left|-\frac{1}{2}\right|\right)$



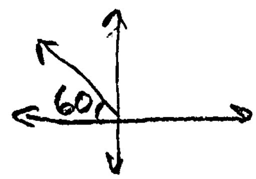
θ in QUAD. II, QUAD III

$\theta' = \cos^{-1}\left(\frac{1}{2}\right)$



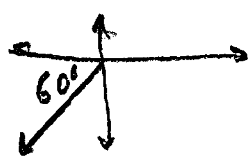
$\therefore\ \theta' = 60^\circ$

QUAD II



$\therefore\ \boxed{\theta = 180 - 60 = 120^\circ}$

QUAD III



$\boxed{\theta = 180 + 60 = 240^\circ}$

$60^\circ = \frac{\pi}{3}$ $\therefore\ \theta_{II} = \pi - \frac{\pi}{3} = \boxed{2.0944}$

$\theta_{III} = \pi + \frac{\pi}{3} = \boxed{4.1888}$

$\boxed{\text{or } \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}}$ EXACT

③ $2\cos^2\theta - \cos\theta - 1 = 0$

Quadratic Form

Let $x = \cos\theta$

Ch5-4

$$2x^2 - x - 1 = 0$$

sum $= -1$

prod $= -2$

$+1 \quad -2$

$$(2x^2 - 2x) + (x - 1) = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(2x+1)(x-1) = 0$$

SPLIT INTO 2 EQUATIONS

SUBSTITUTE $x = \cos\theta$

$$2\cos\theta + 1 = 0$$

$$\cos\theta = -\frac{1}{2}$$

S A
T C

∴ QUAD II, III

$$\theta' = \cos^{-1}\left(\left|-\frac{1}{2}\right|\right) = 60^\circ$$

∴ $\boxed{\theta = 120^\circ}$ $\boxed{\theta = 240^\circ}$

or $\boxed{\theta = \frac{2\pi}{3}}$ $\boxed{\theta = \frac{4\pi}{3}}$

$$\cos\theta - 1 = 0$$

$$\cos\theta = 1$$

QUAD I, IV BUT 1 ⇒ ∴ QUADRANTAL

BUT

(0,1)
(-1,0)
(1,0)
(0,-1)

$\boxed{\cos 0^\circ = 1}$ $\boxed{\cos 360^\circ = 1}$

$\boxed{\theta = 0^\circ}$ $\cancel{\theta = 360^\circ}$

or $\boxed{\theta = 0}$ $\cancel{\theta = 2\pi}$

LOOK

oops

Primary Solution $0 \le x < 360$

(4) $\tan^2 x + 4\tan x = 1$

$\tan^2 x + 4\tan x - 1 = 0$ (Let $X = \tan x$)

$X^2 + 4X - 1 = 0$

sum $= 4$, prod $= -1$: NOT FACTORABLE, use QUADRATIC FORMULA

$a = 1 \qquad b^2 - 4ac$

$b = 4 \qquad (4)^2 - 4(1)(-1)$

$c = -1 \qquad 16 + 4 = 20 = d$ NOT Perfect Square $\Rightarrow$ NOT FACTORABLE ✓

$$X = \frac{-b \pm \sqrt{d}}{2a} = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2}$$

$X = -2 \pm \sqrt{5}$ (SUBSTITUTE $X = \tan x$)

$X = -2 + \sqrt{5}$	$X = -2 - \sqrt{5}$
$\tan x = -2 + \sqrt{5}$	$\tan x = -2 - \sqrt{5}$
$\tan^{-1}(\lvert -2 + \sqrt{5} \rvert) = x'$	$\tan^{-1}(-2 - \sqrt{5}) = x'$
$\tan^{-1}(.2361) = x'$	$\tan^{-1}(\lvert -4.2361 \rvert) = x'$
$\tan = (+)$; S A / T C; QUAD I, III	QUAD II, IV since $\tan$ (−)
$x' = 13.284^\circ \sim .2319$ rad	$x' = 76.7175^\circ \sim 1.3390$ rad
QI $\boxed{x = 13.3^\circ, .2319 \text{ rad}}$	QII $\boxed{x = 103.3^\circ, 1.8026 \text{ rad}}$
QIII $\boxed{x = 193.3^\circ, 3.3734 \text{ rad}}$	QIV $\boxed{x = 283.3^\circ, 4.9442 \text{ rad}}$

See Wolfram Alpha plot next pg. LOOK ⟹

WolframAlpha computational knowledge engine

Solve tan^2(x) + 4tan(x) -1 = 0 where 0<=x<2*pi

Examples Random

Interpreting as: **tan^2(x) + 4tan(x) -1 = 0 where 0<=x<2*pi**

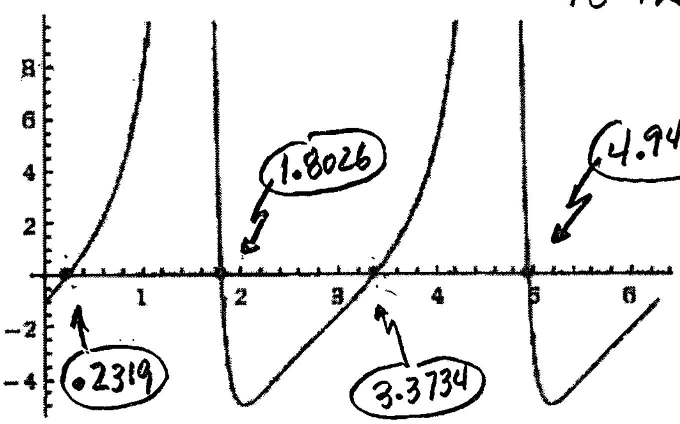
Other queries to try: 2(x) + 4tan(x) -1 = 0 | 2*pi

Input interpretation: corresponding function ⇒ replace 0 with y

plot	$\tan^2(x) + 4\tan(x) - 1 = 0$	$x = 0$ to 2π

when $y = 0$, the solutions to the equation are the X-intercepts!

Root plot:



Computed by Wolfram *Mathematica*

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(39)* $\sin^2 x - \cos^2 x = 0$

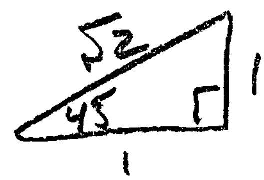
$$\sin^2 x - (1 - \sin^2 x) = 0$$

$$\sin^2 x - 1 + \sin^2 x = 0$$

$$2\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm\sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$



$$\therefore\ x' = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

sin is ± in QI to QIV

$$\therefore\ \boxed{x = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$$

$$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

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* try to put the equation in terms of only the sin or cos.