Mth 113 | Thursday 2-21-13 (Week #7) | Class Notes

Ch. 5-4 Pg 197-198:
Using Identities to help solve a trig. Equation

When a trig. equation involves more than one trig. function — try to rewrite in terms of one function using trig. identities, then solve.

Ex 5-4C Pg 198

① $\tan\theta - \cot\theta = 0$

$\tan\theta - \frac{1}{\tan\theta} = 0$

$\tan^2\theta - 1 = 0$

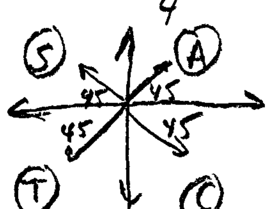
$\tan^2\theta = 1$

$\tan\theta = \pm 1 \Rightarrow$ All Quadrants

$45^\circ = \frac{\pi}{4}$

$\boxed{\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$

$\frac{\pi}{4}, \frac{135}{180}\pi, \frac{225}{180}\pi, \frac{315}{180}\pi$



$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

(2) $2\cos^2 x - 3\sin x - 3 = 0$

$2(1-\sin^2 x) - 3\sin x - 3 = 0$

$2 - 2\sin^2 x - 3\sin x - 3 = 0$

$-2\sin^2 x - 3\sin x - 1 = 0$

$2\sin^2 x + 3\sin x + 1 = 0$ Let $x = \sin x$

$2x^2 + 3x + 1 = 0$

sum = 3
prod = 2

1 2

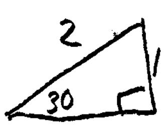
$(2x^2 + 2x) + (1x + 1) = 0$

$2x(x+1) + 1(x+1) = 0$

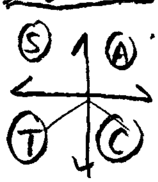
$(2x+1)(x+1) = 0$

$x = \sin x$

$2\sin x + 1 = 0$



$\sin x = -\frac{1}{2}$



$x' = \sin^{-1}(|-\tfrac{1}{2}|) = 30^\circ$

$\sin(-) \Rightarrow$ QIII, QIV

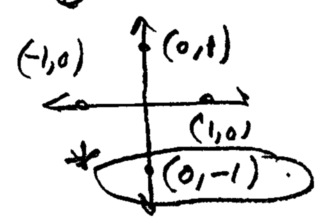
$\therefore \boxed{x = 210^\circ, 330^\circ}$

$x = \frac{21\cancel{0}}{18\cancel{0}}\pi, \ \frac{33\cancel{0}}{18\cancel{0}}\pi$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\sin x + 1 = 0$

$\sin x = -1$ Quadrantal



$\boxed{x = 270^\circ}$

$\boxed{x = \frac{3\pi}{2}}$

Find All solutions, not just primary ($0 \le x < 360°$)
solution to trig. equations — pg 199

"Primary" solutions are only for one cycle (of a sin or cos function). Since the trig. functions are periodic, there are actually an infinite number of solutions.

In general, if you know the periodicity of the function, you assign an integer, usually K or N, to multiply by this periodicity to represent All solutions.

Note: Period of sin is $360°$ or 2π
cos is 360 or 2π
tan is $180°$ or π

Function	Primary Solution	All solutions, K is any integer
$\sin\theta$	$35°$	$35 + 360K$
$\sin\theta$	$\frac{\pi}{4}$	$\frac{\pi}{4} + 2\pi K$
$\cos\theta$	$20°, 135°$	$20 + 360K,\ 135 + 360K$
$\cos\theta$	$\frac{\pi}{6}$	$\frac{\pi}{6} + 2\pi K$
$\tan\theta$	$45°$	$45° + 180K$
$\tan\theta$	$\frac{\pi}{3}$	$\frac{\pi}{3} + \pi K$

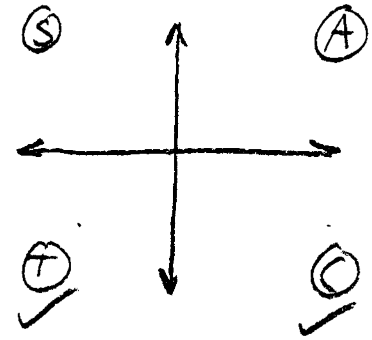
Pg 201

(7)

$\csc\theta + 2 = 0$

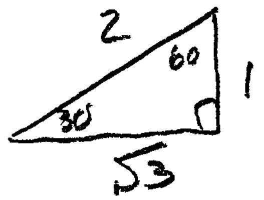
$\Downarrow$

$\frac{1}{\sin}$



$\csc\theta = -2$

$\therefore \frac{1}{\sin\theta} = -2$



$\sin\theta = -\frac{1}{2}$

$\sin^{-1}\left(\left|-\frac{1}{2}\right|\right) = \theta' = 30^\circ$

$\therefore \boxed{\theta = \{210^\circ, 330^\circ\}}$ degrees

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$\overset{7}{\cancel{210}}{}^\circ \cdot \frac{\pi \text{ rad}}{\underset{6}{\cancel{180}}} = \boxed{\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\} = \theta}$

radians

$\overset{11}{\cancel{330}} \cdot \frac{\pi \text{ rad}}{\underset{6}{\cancel{180}}}$

WolframAlpha computational knowledge engine

Solve csc(x)+2=0

Examples Random

Input interpretation:

solve	$\csc(x) + 2 = 0$

$\csc(x)$ is the cosecant function »

Results:

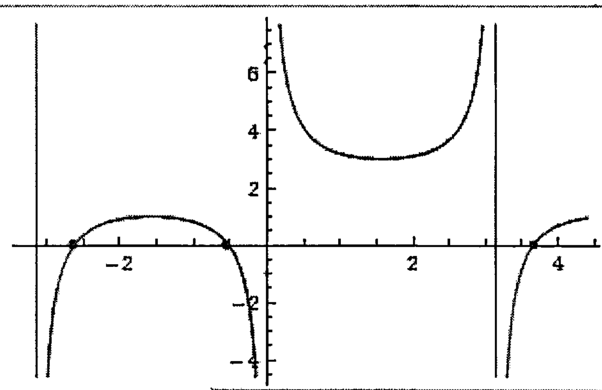
More digits | Step-by-step solution

$x = \pi\left(2n - \frac{1}{6}\right) \approx 3.1416\,(2.0000\,n - 0.16667)$ and $n \in \mathbb{Z}$ ✓

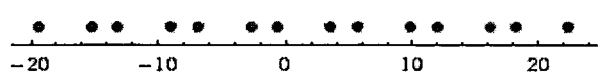
$x = \pi\left(2n + \frac{7}{6}\right) \approx 3.1416\,(2.0000\,n + 1.1667)$ and $n \in \mathbb{Z}$ ✓

$\mathbb{Z}$ is the set of integers »

Root plot:



Number line:



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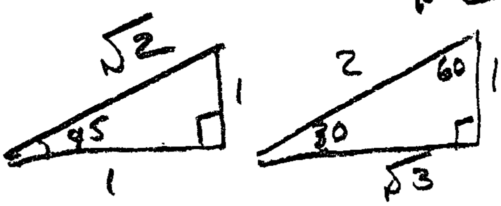
$$(2\cos^2\theta - 1)(\cot\theta - 1) = 0$$

$$2\cos^2\theta - 1 = 0$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\sqrt{\frac{1}{2}}$$ All 4 QUAD's



$$\cos^{-1}\left(\left|\frac{1}{2}\right|\right) = \theta' = 45^\circ$$

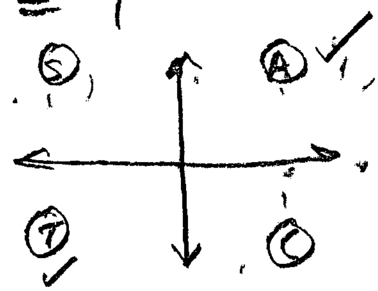
$\therefore$

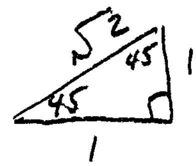
$$\boxed{\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ}$$

$$\cot\theta - 1 = 0$$

$$\cot\theta = 1$$

$$\frac{1}{\tan\theta}$$





QI, QIII, $\theta' = 45^\circ$

$$\boxed{\theta = 45^\circ, 225^\circ}$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$