Mth113 | Weds. 2-27-13 | Class Notes

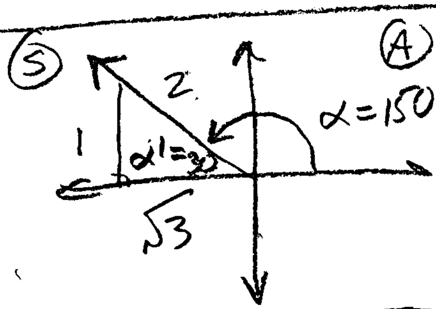
Use Half-Angle Identities:

Pg 190 Exact $\sin\theta$, $\cos\theta$, $\tan\theta$

(43) $\theta = 75^\circ$ or $\frac{5\pi}{12}$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \Big| \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{\cos\theta} \text{ or } \frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$$



$$\therefore \sin\left(\frac{150}{2}\right) = \sin 75^\circ$$

$$\sin\left(\frac{150}{2}\right) = +\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{\frac{2}{1}}}$$

$$= \sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}} = \sqrt{\frac{2}{4}+\frac{\sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2} = \frac{1}{2}\sqrt{2+\sqrt{3}}}$$

43 (CONT)

$$\cos\left(\frac{150}{2}\right) = +\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}} = \sqrt{\frac{2}{4} - \frac{\sqrt{3}}{4}}$$

$$= \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$

$$\tan 75^{\circ} = \frac{\sin 75}{\cos 75}$$

$$= \frac{\frac{\sqrt{2+\sqrt{3}}}{2}}{\frac{\sqrt{2-\sqrt{3}}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$$

$$\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}} \cdot \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}\,\sqrt{2-\sqrt{3}}}{2-\sqrt{3}} \cdot \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

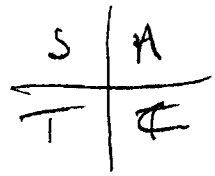
$$\frac{\sqrt{2+\sqrt{3}}\,\sqrt{2-\sqrt{3}}\,(2+\sqrt{3})}{1}$$

$$\sqrt{(2+\sqrt{3})(2-\sqrt{3})}\,(2+\sqrt{3}) = \left(\sqrt{4-3}\right)(2+\sqrt{3})$$

$$\boxed{2+\sqrt{3}}$$

(57) $\tan x = 1$ $\boxed{x = 45^\circ}, \cancel{135^\circ}, \boxed{225^\circ}, \cancel{315^\circ}$

PG 202



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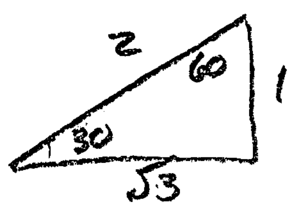
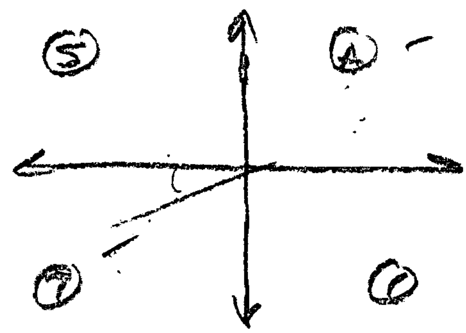
(77) $\sqrt{3}\tan\left(\frac{\theta}{4}\right) = 1$

PG 202

$$\tan\left(\frac{\theta}{4}\right) = \frac{1}{\sqrt{3}}$$

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\theta}{4}$$





$$\theta' = 30^\circ$$

$$\therefore \frac{\theta}{4} = 30^\circ, \quad \frac{\theta}{4} = 210^\circ$$

$$\theta = 120^\circ, \quad \theta = 840^\circ$$

$$\boxed{\begin{array}{ll} \theta = 120^\circ + \cancel{360}K & K = \text{Integer} \\ \theta = 840^\circ + \cancel{360}K & \end{array}}$$

$$* \quad \boxed{\theta = 120 + 720K \quad K = \text{Any integer}}$$

(88) $\sin^2\left(\frac{\theta}{2}\right) = \cos\theta$

MISTAKES AND ALL :(
Mr. C

pg 202

$1 - \cos^2\left(\frac{\theta}{2}\right) = \cos\theta$

D.A. cos

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos^2\theta = \cos 2\theta + \sin^2\theta$

$\cos^2\left(\frac{\theta}{2}\right) + \cos\theta - 1 = 0$

DA IDEN.

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\left(\cos\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}\right) + \cos\theta - 1 = 0$

$(2\cos^2\theta - 1)(2\cos^2\theta - 1) + \cos\theta - 1 = 0$

$4\cos^4\theta - 4\cos^2\theta \cancel{+1} + \cos\theta \cancel{-1} = 0$

$4\cos^4\theta - 4\cos^2\theta + \cos\theta = 0$

$4\cos^2\theta(\cos^2\theta - 1) + \cos\theta = 0$

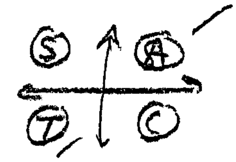
START OVER

Not getting anywhere, lets try a half-angle formula instead of double-angle... see next pg.

88

pg 202

$$\sin^2\left(\frac{\theta}{2}\right) = \cos\theta$$



$$\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \cos\theta$$

$$\boxed{\pm\sqrt{\frac{1+\cos\alpha}{2}} = \cos\frac{\alpha}{2}}$$

$$\therefore \sqrt{\frac{1+\cos\theta}{2}} \cdot \sqrt{\frac{1+\cos\theta}{2}} = \cos\theta$$

$$\frac{1+\cos\theta}{2} = \cos\theta$$

$$1+\cancel{\cos\theta} = 2\cos\theta$$

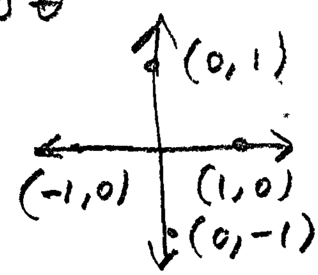
$$-\cancel{\cos\theta} \qquad -\cos\theta$$

$$\cos\theta = 1$$

$$\therefore \boxed{\theta = 0^\circ}$$

Good Approach,

Wrong! Answer



Didn't match book answer, mistake was accidently using the cos half-angle formula!

Arrrg.

Should be $\pm\sqrt{\frac{1-\cos\alpha}{2}}$!!

START over

88 (CONT)

$$\sin^2\left(\frac{\theta}{2}\right) = \cos\theta$$

$$\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = \cos\theta$$

Use $\sin\left(\frac{\theta}{2}\right)$ half-angle identity

$$\sqrt{\frac{1-\cos\theta}{2}} \cdot \sqrt{\frac{1-\cos\theta}{2}} = \cos\theta$$

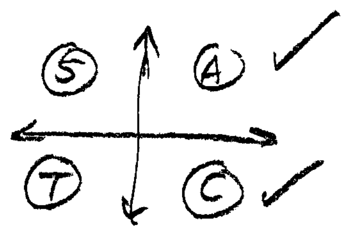
$$\frac{1-\cos\theta}{2} = \cos\theta$$

$$1-\cos\theta = 2\cos\theta$$

$$+\cos\theta \qquad +\cos\theta$$

$$1 = 3\cos\theta$$

$$\therefore \cos\theta = \frac{1}{3}$$



COS POSITIVE

$$\cos^{-1}\left(\left|\tfrac{1}{3}\right|\right) = \theta'$$

$$70.53^\circ = \theta'$$

$$\therefore \theta = 70.53^\circ,\ 360 - 70.53^\circ$$

$$\boxed{\theta = 70.5^\circ,\ 289.5^\circ}$$

1.23 rad, 5.05 rad

Finally, got it! :)