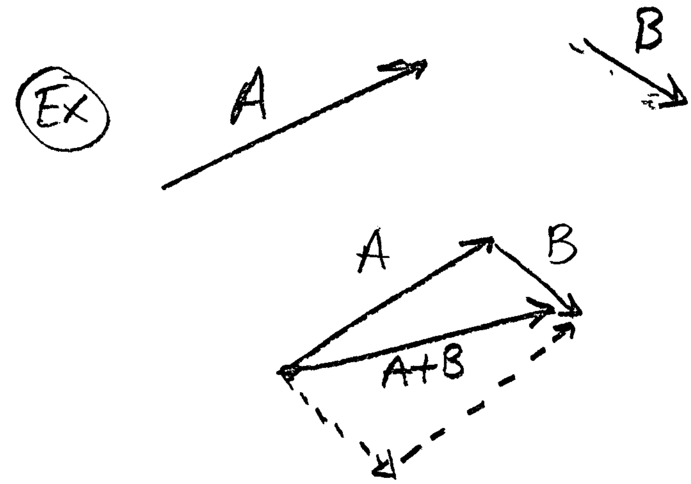
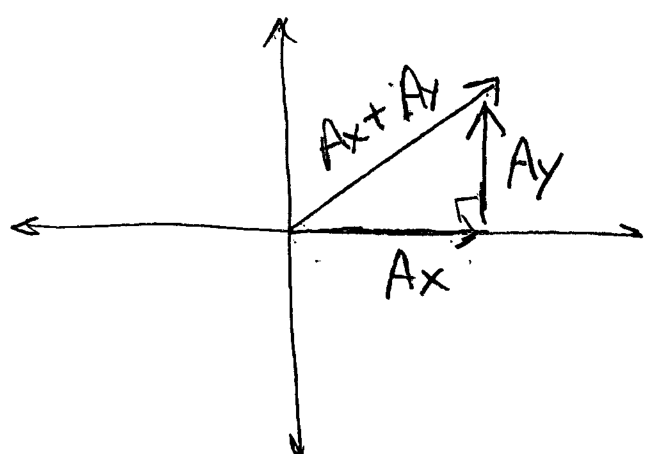
Mth 113 | Tuesday 3-12-13 | Class Notes

Adding vectors: one way to add two vectors is to combine them into a parallelogram tail-to-head. The sum is called the resultant vector.

(Ex)



(Ex)



If we know the rectangular coordinates of two (or more) vectors, the resultant vector is (sum of the Ax's, sum of the Ay's)

horizontal components — vertical components

Ex ① $A = (-3, 5)$, $B = (1, 3)$ $A+B = ?$

pg 230

$\boxed{A + B = (-2, 8)}$

Ex 2 $A = (2, -\frac{1}{2})$, $B = (-6, 3\frac{1}{2})$, $C = (3, -2)$

$\boxed{A + B + C = (-1, 1)}$

If you are given vectoo in polar coordinates, and you want their sum in polar coordinates (which you usually will), convert to rectangular, add x components, y components, then convert back to polar

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Ex 3 $V = (13.8, 40.2^\circ)$, $W = (20.9, 164.6^\circ)$, sum?

$V_x = 13.8 \cos 40.2$ $\quad W_x = 20.9 \cos 164.6$

$= 10.540$ $\quad + \quad$ $= -20.150$

$V_y = 13.8 \sin 40.2$ $\quad W_y = 20.9 \sin 164.6$

$= 8.907$ $\quad + \quad$ $= 5.550$

$V + W = (-9.61, 14.457)$

$\therefore |V + w| = \sqrt{(-9.61)^2 + (14.457)^2} = 17.359$

QUAD II $\theta' = \tan^{-1}\left(\frac{14.457}{9.61}\right) = 56.386$ $\therefore \theta = 180 - 56.386$

$\theta = 123.61^\circ$

$\therefore \boxed{V + W = (17.4, 124^\circ)}$

Three forces are acting on a point,
$F_1 = (25\ lb_F, 30^\circ)$, $F_2\ (40\ lb_F, 100^\circ)$, $F_3\ (50\ lb_F, -40^\circ)$
Find resultant Force acting on point. (F_r)

$F_{1x} \Rightarrow 25\cos 30^\circ = 25\left(\frac{\sqrt{3}}{2}\right) = 21.651$

$F_{2x} \Rightarrow 40\cos 100^\circ = 40(-.1736) = -6.946$

$F_{3x} \Rightarrow 50\cos(-40^\circ) = 50\cos(320) = 38.302$

$F_{r_x} = 53.007$

$F_{1y} \Rightarrow 25\sin 30 = 25\left(\frac{1}{2}\right) = 12.500$

$F_{2y} \Rightarrow 40\sin 100 = 40(.9848) = 39.392$

$F_{3y} \Rightarrow 50\sin(-40) = 50(\sin 320) = -32.139$

$F_{ry} = 19.753$

$\therefore F_r = (53.007, 19.753)$

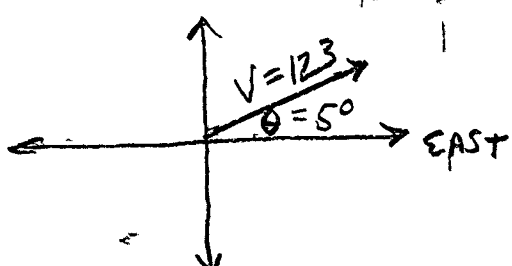
$\Rightarrow |F_r| = \sqrt{(53.007)^2 + (19.753)^2} = 56.568$

QUAD I $\tan^{-1}\left(\frac{19.753}{53.007}\right) = 20.437^\circ = \theta$

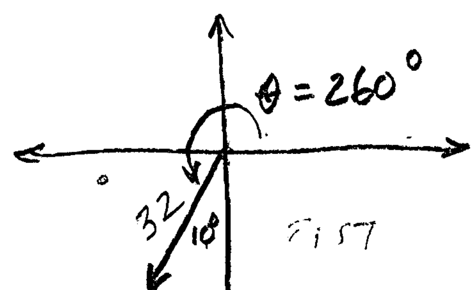
$\therefore \boxed{F_r = (56.6, 20.4^\circ)}$

EX 6-3-d pg 231

Find true course and ground speed of airplane flying (heading) 5° North of East with airspeed of 123 knots. The wind is 32 knows in the direction 10° west of south.



Airplane Heading



Wind Direction

$A(123, 5°)$ + $W(32, 260°)$

$A_x = 123\cos 5°$
$= 122.531$ +

$W_x = 32\cos 260°$
$= -5.557$

$G_x = 116.443$ → 116.974

Opps I made a typo and used 122 − Wx vs 122.531

$A_y = 123\sin 5°$
$= 10.720$ +

$W_y = 32\sin 260°$
$= -31.514$

$G_y = -20.794$

$G = (116.443, -20.794)$

Quad IV

$|G| = \sqrt{(116.443)^2 + (-20.794)^2} = 118.285 = 118$

Book is right 119

$\theta' = \tan^{-1}\left(\frac{20.794}{116.443}\right) = 10.124°$ ∴ $\theta = 360 - \theta'$

$\theta = 349.87$

$\theta = 350°$

∴ $G(118, 350°)$ 119

→ 10° south of east

Zero Vector

Length is 0 so direction does not matter.

$0 = (0, \theta)$ θ is any angle

Opposite Vector

The opposite of a vector has equal length, but opposite direction
↓
$\pm 180^\circ$

If $V = (|V|, \theta)$

$-V = (|V|, \theta \pm 180^\circ)$
↑
same magnitude

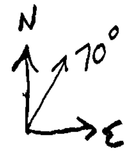
Ex $V = (26, 45^\circ)$

$-V = (26, 225^\circ)$ or $(26, -135^\circ)$
↑
is this the same as $+225^\circ$?

Ex 6-3E pg 232

Airplane: Groundspeed (true) (200 knots, due North)

$(200, 90^\circ)$



Heading (direction pointed) (225 knots, 20° e of N)

$(225, 70^\circ)$

Find Wind speed vector W

$$G = H + W$$

$$\therefore W = G - H \quad \text{or } \underset{\Downarrow}{G} + \underset{\Downarrow}{(-H)}$$

$$(200, 90^\circ) \quad (225, 250^\circ)$$

$$G_x = 200\cos 90^\circ = 0 \quad \Big| \quad -H_x \Rightarrow 225\cos 250^\circ$$

$$\therefore W_x \qquad = 0 \;+\; = -76.954$$

$$W_x = -76.954$$

$$G_y = 200\sin 90 = 200 \quad \Big| \quad -H_y \Rightarrow 225\sin 250^\circ$$

$$\therefore W_y = \qquad = 200 \;+\; = -211.431$$

$$W_y = -11.431$$

$$\therefore W(-76.954, -11.431)$$

Quad III

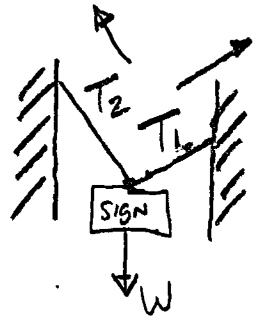
$$|W| = \sqrt{(-76.954)^2 + (-11.431)^2} = 77.798$$

$$\theta' = \tan^{-1}\left(\frac{11.431}{76.954}\right) = 8.449^\circ \therefore \theta = 180 + \theta'$$

$$= 188^\circ$$

$$\boxed{W(77.8, 188^\circ)}$$ Book??

Ex 2
pg 233



T_1 = Force Vector (tension) in wire 1
T_2 = Force Vector (tension) in wire 2
W = weight vector of sign (Force down from gravity)

Physics: for the sign to be stationary,

$$\sum \text{Forces} = 0$$

$$\therefore \underset{\text{know}}{T_1} + \underset{?}{T_2} + \underset{\text{know}}{W} = 0$$

$$T_2 = -T_1 + -W$$

$$T_1\,(400, 45^\circ) \therefore -T_1\,(400, 225^\circ)$$
$$W\,(800, 270^\circ) \therefore -W\,(800, 90^\circ)$$

$$-T_{1x} = 400\cos 225 \qquad -W_x = 800\cos 90^\circ = 0$$
$$T_{2x} = -282.843 + 0$$

$$-T_{1y} = 400\sin 225 \qquad -W_y = 800\sin 90^\circ$$
$$T_{2y} = -282.843 + = 800$$
$$T_{2y} = 517.157 \qquad \therefore T_2\,(-282.843, 517.157)$$
QUAD II

$$|T_2| = \sqrt{(-282.843)^2 + (517.157)^2} = 589.450$$

$$\theta' = \tan^{-1}\left(\frac{517.157}{282.843}\right) = 61.32^\circ \therefore \theta = 180 - \theta'$$
$$\theta = 118.68^\circ$$

$$\boxed{T_2\,(589.\text{lb}_F,\ 119^\circ)}$$

EX 2 (CONT)