Mth 113 | Weds. 3-13-13 | Class Notes

Ch. 7-1 Complex Numbers and Polar Coordinates

$i = \sqrt{-1}$ $\therefore$ $\boxed{i^1 = i \quad i^3 = -i \quad i^2 = -1 \quad i^4 = 1}$

i = imaginary

Rectangular Form of a Complex Number

$\boxed{a + bi}$

real part, imaginary part

a, b are real numbers

$a=0$, pure imaginary

$b=0$, pure real

$a+bi$, $a-bi$ complex conjugates

$a+bi = c+di$ iff $a=c$, $b=d$

Complex numbers are not ordered,

$\Rightarrow$ you can't say one is $<$ or $>$ another.

Add/subtract complex numbers:

Ex $(5+4i)+(3+2i) = \boxed{8+6i}$

Multiply complex numbers:

Ex $(5+4i)(3+2i) = 15+10i+12i+8i^2$

$= 15+22i-8$

$= \boxed{7+22i}$

Divide complex numbers:
(multiply by complex conjugate of denominator)

Ex $\dfrac{5+4i}{3+2i} \cdot \dfrac{3-2i}{3-2i} = \dfrac{15-10i+12i-8i^2}{9-4i^2}$

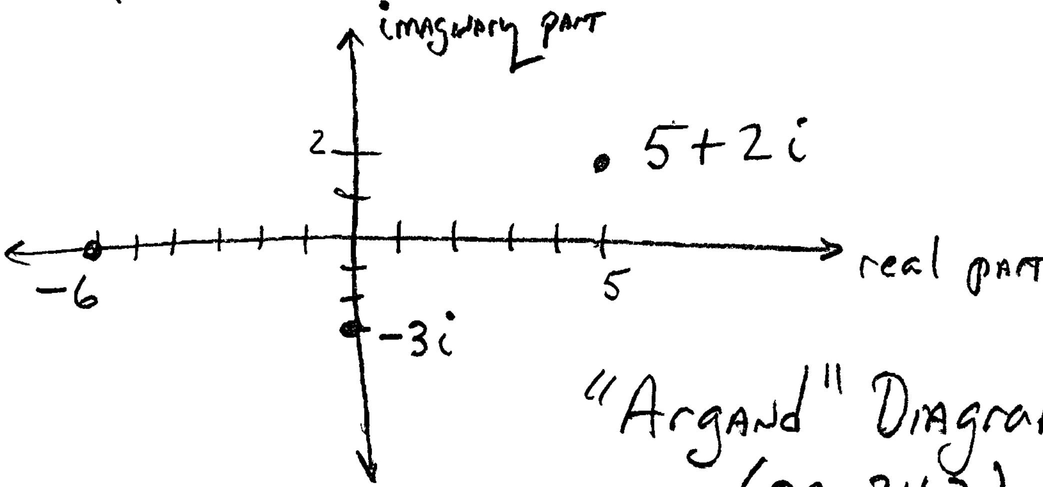
$= \dfrac{23+2i}{9+4}$

$= \dfrac{23+2i}{13}$

$= \boxed{\dfrac{23}{13}+\dfrac{2}{13}i}$

write in the correct complex form $a+bi$ (rectangular)

Complex Coordinate PLANE



"Argand" Diagram
(pg 243)

EX 7-1F
pg 243

Alternating Current

$$\underset{\text{impedance}}{Z} = \underset{\substack{\text{Resistance}\\ \text{(real part)}}}{R} + \underset{\substack{\text{reactance}\\ \text{(imaginary part)}}}{X i}$$

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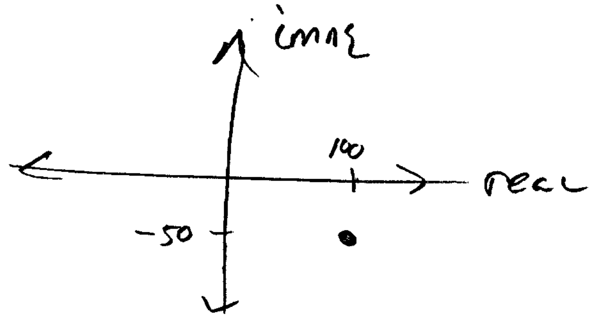
resistors

coils ⇒ resist Δ current
capacitors ⇒ resist Δ voltage

Graph $Z = 100 - 50i$

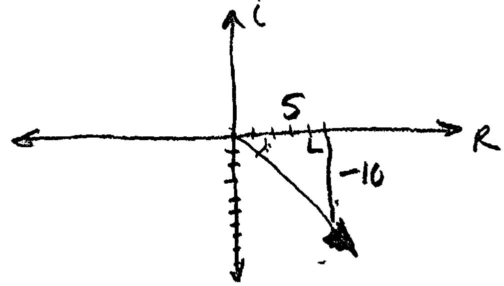


Polar Form of a Complex Number

$z =$

EX 7-1G ① Convert $5 - 10i$ to polar form $z = ?$

Pg 245



CALLED THE MODULUS

$r = \sqrt{5^2 + (-10^2)} = \sqrt{125} = 5\sqrt{5}$

$\theta' = \tan^{-1}\left(\frac{10}{5}\right) = 63.4^\circ \therefore \theta = +296.6^\circ$ or -63.4°

real part $= z_x = 5\sqrt{5}\cos(-63.4^\circ)$

imag. part $= z_y = 5\sqrt{5}\sin(-63.4^\circ)$

$\therefore z = 5\sqrt{5}\cos(-63.4^\circ) + [5\sqrt{5}\sin(-63.4^\circ)]i$

* SHORTHAND WAY OF WRITING $\boxed{z = 5\sqrt{5}\,\mathrm{cis}(-63.4^\circ)}$

where "cis" means $\cos\theta + i\sin\theta$

WolframAlpha computational knowledge engine

convert to polar form 5-10i

Examples Random

Assuming i is the imaginary unit | Use i as a variable instead

Input interpretation:

convert $5 - 10i$ to polar form

i is the imaginary unit »

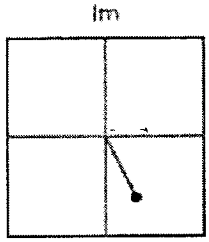
Result:

Exact form

$r \approx 11.1803$ (radius), $\theta \approx -63.4349°$ (angle)

APPROXIMATE

Position in the complex plane:



Computed by Wolfram Mathematica

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$5\sqrt{5} \approx 11.1803$

$$\boxed{z = 11.1803\,\mathrm{cis}(-63.4°)}$$

$z(11.1, -63.4)$

WolframAlpha computational.. knowledge engine

convert to polar form 5-10i

Examples | Random

Assuming i is the imaginary unit | Use i as a variable instead

Input interpretation:

convert $5 - 10\,i$ to **polar form**

i is the imaginary unit »

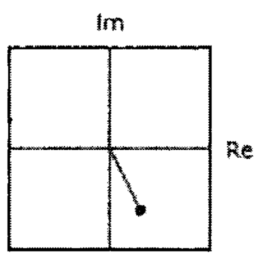
Result: | Approximate form

$$r = 5\sqrt{5} \text{ (radius)}, \quad \theta = -\frac{180 \tan^{-1}(2)}{\pi}{}^{\circ} \text{ (angle)}$$

$\tan^{-1}(x)$ is the inverse tangent function »

EXACT

Position in the complex plane:



EX2 Convert to Rectangular Form

PG 245

$$z = 5 \operatorname{cis} 150^\circ$$

$$z_x = 5\cos 150 = -4.3301$$

$$z_y = 5\sin 150 = 2.5$$

$$\therefore \boxed{z = -4.3 + 2.5i}$$

APPROXIMATE

NOTE: Since the reference angle for 150° is 30° you could have found the EXACT values of the $\sin 30^\circ$, $\cos 30^\circ$ $\Rightarrow$

$$z = 5\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$\boxed{z = -\frac{5\sqrt{3}}{2} + \frac{5}{2}i}$$

EXACT