Mth 113 | Wednesday 3-20-13 | Class Notes

Ch. 7-1 Cont. (see pg. 246)

* Complex Numbers are easy to add in rectangular form, just combine the real parts and imaginary parts. Ex $(3+5i)+(4+6i)$
$= 7+11i$

This is similar to the process for adding vectors in the x-y plane
$\Rightarrow$ ADD the x components, add y components.

This should not be a surprise because in the complex plane, complex numbers are quite similar to vectors.

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* Complex numbers are easy to multiply in polar form, and divide.

Note: you can multiply vectors $\Rightarrow$ dot product (scalar)
$\Rightarrow$ cross-product (vector) (area of ▱)

Multiplying Complex Numbers in Polar Form.

$$\boxed{(r_1 \operatorname{cis} \theta_1)(r_2 \operatorname{cis} \theta_2) = \underbrace{r_1 r_2}_{\text{MULTIPLY MODULI}} \operatorname{cis}(\underbrace{\theta_1 + \theta_2}_{\text{ADD Angles}})}$$

7-1-H
EX1
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① $(2 \operatorname{cis}(110°))(10 \operatorname{cis}(300°))$

$= 20 \operatorname{cis}(410°) = \boxed{20 \operatorname{cis}(50°)}$

② $V = I\,Z$

VOLTAGE (CURRENT)(impedance)

resistance is a DC circuit. OR AN AC WITH ONLY resistors.

$V = (\underbrace{4 \operatorname{cis} 30°}_{\text{AMPS}})(\underbrace{2 \operatorname{cis} 15°}_{\text{OHMS}})$

$= \boxed{8 \operatorname{cis}(45°)}$ VOLTS

Dividing Complex Numbers in Polar Form

$$\boxed{\frac{r_1 \operatorname{cis} \theta_1}{r_2 \operatorname{cis} \theta_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)} \quad r_2 \neq 0$$

DIVIDE the moduli

SUBTRACT THE Angles

7-1-H

EX3

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$$\frac{15 \operatorname{cis} 30^\circ}{18 \operatorname{cis} 80^\circ} = \frac{15}{18} \operatorname{cis}(-50^\circ)$$

$$= \boxed{\frac{5}{6} \operatorname{cis}(-50^\circ)}$$

$$\text{or } \frac{5}{6} \operatorname{cis}(310^\circ)$$

The process for multiplying powers of $r\,\text{cis}\,\theta$ produces a pattern called $\boxed{\text{De Moivre's Theorem}}$

$$(r\,\text{cis}\,\theta)^1 = r\,\text{cis}\,\theta$$

$$(r\,\text{cis}\,\theta)^2 = (r\,\text{cis}\,\theta)(r\,\text{cis}\,\theta) = r^2\text{cis}(\theta+\theta)$$

$$= r^2\text{cis}(2\theta)$$

$$(r\,\text{cis}\,\theta)^3 = r^2\text{cis}(2\theta)\,r\,\text{cis}(\theta) = r^3\text{cis}(3\theta)$$

$$\vdots \qquad \text{De Moivre's Theorem} \qquad \vdots$$

$$\boxed{(r\,\text{cis}\,\theta)^N = r^N\text{cis}(N\theta)}$$

For any real number N

Ex $(5\,\text{cis}\,137^\circ)^3 = 5^3\text{cis}(3 \cdot 137)$

$$= 125\,\text{cis}(411^\circ)$$

$$= \boxed{125\,\text{cis}(51^\circ)}$$

If you want to raise a complex number that is in rectangular form to a power,

convert to polar $\Rightarrow$ use DeMoivre's Theorem $\Rightarrow$ convert result back to rectangular

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Ex 2 $(1+0.8i)^6$ Nearest tenth

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$|r| = \sqrt{1^2 + (.8)^2} = \sqrt{1.64} = 1.2806$

$QI \Rightarrow \theta = \theta' \Rightarrow \tan^{-1}\left(\frac{.8}{1}\right) = \tan^{-1}(.8) = 38.660°$

$\therefore \left(1.2806 \operatorname{cis}(38.660°)\right)^6$

$\Rightarrow (1.2806)^6 \operatorname{cis}(6 \cdot 38.660) = 4.410 \operatorname{cis}(231.96°)$

Back to rectangular

$\Rightarrow 4.410\cos(231.96) + 4.410\sin(231.96)\, i$

$-2.717 - 3.473i$

$\boxed{-2.7 - 3.5i}$

Extending De Moivre's Theorem

to roots, recall $x^{\frac{1}{N}} = N^{th}$ root of x

(of complex numbers) (Ex) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$$z = r\,\text{cis}\,\theta$$

$$z^{\frac{1}{N}} = r^{\frac{1}{N}}\,\text{cis}\left[\frac{1}{N}\theta + \frac{1}{N}(360K)\right] \quad 0 \le K < N$$

K, N are positive integers

principle root $\Rightarrow K = 0$

$\Rightarrow r^{\frac{1}{N}}\,\text{cis}\left[\frac{1}{N}\theta\right]$

ALL real numbers can be written in complex form $\Rightarrow$ they have N roots

7-1-J

Ex 1 ① Find the 3 cube roots of 1

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$$1 = 1\,\text{cis}(0^\circ)$$

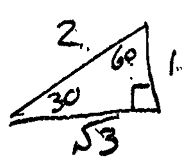
$$\therefore \left[1\,\text{cis}(0^\circ)\right]^{\frac{1}{3}} = 1^{\frac{1}{3}}\text{cis}\left[\frac{1}{3}(0^\circ) + \frac{1}{3}(360K)\right]$$

($1 \rightarrow r$)

$K = 0, 1, 2$

$\Rightarrow$ 3 roots

S	A
T	C



$K=0 \Rightarrow \text{cis}(0^\circ) = \cos(0^\circ) + \sin(0)\,i$

$\boxed{1 + 0i}$

$K=1 \Rightarrow \text{cis}\left(\frac{360}{3}\right) = \cos(120^\circ) + \sin(120^\circ)\,i$

$\boxed{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}$

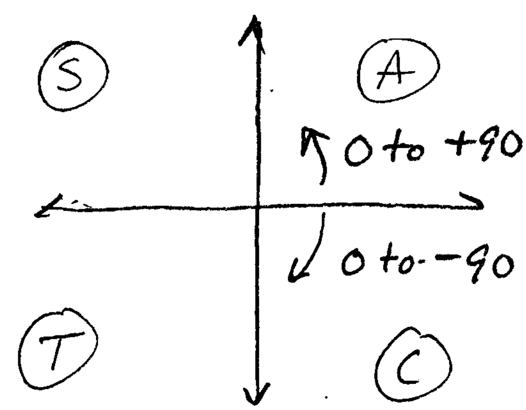
$K=2 = \text{cis}\left(\frac{720}{3}\right) = \cos(240^\circ) + \sin(240)\,i$

$\boxed{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$

CAUTION

A warning about $\tan^{-1}(x)$

The arctan, or $\tan^{-1}$ function has a restricted domain, the allowable values for the angle are $\pm 90^\circ \Rightarrow$ Quadrant I
Quadrant IV



So if, for example, you have a vector $(-9.6, 14.5)$ and you take $\tan^{-1}\left(\frac{14.5}{-9.6}\right) = -56.5^\circ$, you get a QIV angle vs the QII to want.

$\Rightarrow$ take $\tan^{-1}\left(\left|\frac{14.5}{9.6}\right|\right) = 56.5^\circ$ = ref. angle

then compute $\theta \Rightarrow 180 - 56.5 = 123.5^\circ$

Quiz 3 Key

$T_1 + T_2 + W = 0$ $\qquad W(650, 270°)$

$\therefore T_1 = -T_2 - W$ $\qquad T_2(456, 63°)$

$-W \Rightarrow (650, 90°)\ ,\ -T_2(456, 243°)$

$-W_x \Rightarrow 650 \cos 90° = 0$

$-T_{2x} \Rightarrow 456 \cos 243° = -207.02$

$\therefore T_{1x} = -207$

$-W_y \Rightarrow 650 \sin 90 = 650$

$-T_{2y} \Rightarrow 456 \sin 243° = -406.30$

$\therefore T_{1y} = 243.70$

Rectangular

$\therefore \boxed{T_1(-207.0, 243.7)} \Rightarrow$ Quad. II

$|T_1| \Rightarrow \sqrt{(207)^2 + (243.7)^2} = 319.75$

$\theta' = \tan^{-1}\left(\frac{243.7}{207}\right) = 49.66° \quad \therefore \theta = 180 - 49.66$

$\theta = 130.3°$

Polar

$\therefore \boxed{T_1(320, 130°)}$