MTH113 | Thursday 3-21-13 | Class Notes

Ex 7-1-I
2
Pg 249

Find the four 4^{th} roots of $10 - 2\sqrt{39}\, i$ (nearest tenth)

Convert to polar form:

$$r = \sqrt{10^2 + (2\sqrt{39})^2} = \sqrt{100+156} = 16$$

$$\text{QIV } \theta' \Rightarrow \tan^{-1}\left(\left|\frac{2\sqrt{39}}{10}\right|\right) \approx \tan^{-1}(1.2490)$$

$$\theta' = 51.318^\circ \quad \therefore \theta = 360 - \theta' = 308.68^\circ$$

Set $Z = 16\cos(308.68^\circ) + 16\sin 308.68^\circ\, i$

$$Z^{\frac{1}{N}} = r^{\frac{1}{N}} \operatorname{cis}\left(\frac{\theta}{N} + \frac{360^\circ K}{N}\right), 0 \le K < N$$

$N=4$
$K=0,1,2,3$

$$\therefore Z^{\frac{1}{4}} = 16^{\frac{1}{4}} \operatorname{cis}\left(\frac{308.68}{4} + \frac{360K}{4}\right), 0 \le K < 4$$

$$K=0 \Rightarrow Z^{\frac{1}{4}} = 2\cos(77.17^\circ) + 2\sin(77.17^\circ)\, i$$

$$= .4441 + 1.9500\, i$$

$$= \boxed{.4 + 2.0\, i}$$

$$K=1 \Rightarrow 2\operatorname{cis}\left(\frac{308.64}{4} + \frac{360(1)}{4}\right) = 2\operatorname{cis}(167.17^\circ)$$

$$Z^{\frac{1}{4}} = 2\cos(167.17) + 2\sin(167.17)\, i$$

$$= -1.9500 + .4441\, i$$

$$= \boxed{-2.0 + .4\, i}$$

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$K=2 \Rightarrow 2\,\text{cis}\left(\frac{308.64}{4}+\frac{360(2)}{4}\right)=2\,\text{cis}(257.17)$

$z^{\frac{1}{4}}=2\cos(257.17)+2\sin(257.17)\,i$

$=-.4441-1.9500\,i$

$=\boxed{-.4-2.0\,i}$

$K=3 \Rightarrow 2\,\text{cis}\left(\frac{308.64}{4}+\frac{360(3)}{4}\right)=2\,\text{cis}(347.17^\circ)$

$z^{\frac{1}{4}}=2\cos(347.17)+2\sin(347.17)\,i$

$=1.9500-.4441\,i$

$=\boxed{2.0-.4\,i}$

$$\therefore \boxed{\begin{array}{l} z=10-2\sqrt{39}\,i \\ z^{\frac{1}{4}}=\{.4+2.0\,i,\ -2.0+.4\,i,\ \ldots \\ \qquad -.4-2.0\,i,\ 2.0-.4\,i\} \end{array}}$$