Exploring Exponential and Logarithmic Functions

Real Exponents and Exponential Functions

Exponential Function	An equation of the form $y = a \cdot b^{x}$, where $a \neq 0, b > 0$, and $b \neq 1$, is called an exponential function with base b.

Property of Equality for Exponential Functions	Suppose b is a positive number other than 1. Then $b^{x_1} = b^{x_2}$ if and only if $x_1 = x_2$.

Ex.

$2^6 = 2^{3n+1}$

$6 = 3n + 1$

$\frac{5}{3} = n$

Logarithms and Logarithmic Functions

logarithm -The exponent to which a fixed base must be raised in order to obtain a given number.

Suppose $p > 0$ and $b \neq 1$. For $n > 0$, there is a number p such that $\log_b n = p$ if an only if $b^p = n$.

logarithmic function - An equation of the form $y = \log_b x$, where $b > 0$, and $b \neq 1$.

Ex. $\log_5 125 = 5$

Property of Equality for Logarithmic Functions	Suppose $b > 0$ and $b \neq 1$. Then $\log_b x = \log_b x_2$ if an only if $x_1 = x_2$.

Properties of Logarithms

Product Property of Logarithms	For all positive numbers m, n, and b, where $b \neq 1$, $\log_b mn = \log_b m + \log_b n$.
Quotient Property of Logarithms	For all positive numbers m, n, and b, where $b \neq 1$, $\log_b \frac{m}{n} = \log_b m - \log_b n$.
Power Property of Logarithms	For any real number p and positive numbers m and b, where $b \neq 1$, $\log_b m^p = p \cdot \log_b m$.

Common Logarithms

common logarithms - Logarithms to the base 10. **mantissa** - The logarithm of a number between 1 and 10.

characteristic - The integer used to express a base 10 logarithm as the sum of and integer and a positive decimal.

antilogarithm - If $\log x = a$, then $x = \text{antilog } a$.

Natural Logarithms

exponential growth - When a quantity increases exponentially.

natural logarithms - Logarithms to the base e.(e is approximately 2.71828182846)

Solving Exponential Equations

exponential growth rate - The positive constant k in the growth equation $P(t) = P_0 e^{kt}$.

exponential equations - An equation in which variables occur in exponents.

Change of Base Formula	For all possible numbers a, b, and n, where $a \neq 1$ and $b \neq 1$, $\log_a n = \frac{\log_b n}{\log_b a}$

Growth and Decay

general formula for growth and decay - The formula is $y = ne^{kt}$, where y is the final amount, n is the initial amount, k is a constant, and t is the time.

Ex.

$y = ne^{kt}$

$2 = 1e^{k(20)}$

$2 = e^{20k}$

$\ln 2 = \ln e^{20k}$

$\ln 2 = 20k \ln e$

$\ln 2 = 20k$

$\frac{\ln 2}{20} = k$

$0.0347 \approx k$

Δ **Area of a Triangle** - The area of a triangle having vertices at (a,b), (c,d), and (e,f) is $|A|$, where $A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$

Δ $A = \frac{1}{2} bh$

Δ

Hero's Theorem - If s = semi-perimeter (perimeter divided by 2), and a, b, and c represent sides, then $A = \sqrt{s(s-a)(s-b)(s-c)}$ units^2

$A = \frac{ac \operatorname{Sin} B}{2}$ Δ

$\left(\frac{1}{2}\right)$(prod. of two sides)(Sin of enclosed Angle)