# Sequences and Series

## Arithmetic Sequences

**Sequence-** A list of numbers in a specific order
**Term-** Each number in a sequence
**Arithmetic Sequence-** A sequence in which each term after the first is found by adding a constant, called the common difference *d* to the previous term.
**Common Difference-** The number added to find the next term of an arithmetic sequence.
The **formula for finding the nth term of an arithmetic sequence** is $\mathbf{a_n = a_1 + (n-1)d}$ Ex: Find *n* if $a_1 = 1000$, $d = 97$, and $a_n = 2649$

Write the equation→ $a_n = a_1 + (n-1)d$
Set up the equation→ $2649 = 1000 + (n-1)97$
Solve for n→ $1649 = 97n - 97$
$1746 = 97n$
$n = 18$

## Arithmetic Series

**Series-** The indicated sum of the terms of a sequence
**Arithmetic Series-** The indicated sum of the terms of an arithmetic sequence

**Sum of an Arithmetic Series-** $\mathbf{S_n = (n/2)(a_1 + a_n)}$ **and** $\mathbf{S_n = (n/2)[2a_1 + (n-1)d]}$ Finding the sum of Arithmetic Series→ Find $S_n$ if a1 = 70, d = 21, and n = 40
**Sigma or Summation Notation-**

$S_n = (n/2)[2a_1 + (n-1)d]$
$S_{40} = (40/2)[2(70) + (40-1)(-21)]$
$S_{40} = 20(140 - 819)$
$S_{40} = 20(-679)$→ -13580

**Index of Summation-** The variable defined below the in sigma notation
Example of using Sigma Notation: Find the sum of (2k + 5)

Sn = (n/2)(a1 + an)
N = 5, a1 = 11
an = a5 = 2(7) + 5 or 19
Sn = (5/2)(11 + 19)→ 75

## Geometric Sequences

**Geometric Sequence-** A sequence in which each term after the first is found by multiplying the previous term by a constant called the common ratio, *r*.
**Common Ratio-** The number by which each term, after the first, in a geometric sequence is multiplied to obtain the next term
**Geometric means-** The terms between any two nonconsecutive terms in a geometric sequence.
The **Formula for the nth term of a Geometric Sequence** is $\mathbf{a_n = (a_{n-1})(r)}$ **or** $\mathbf{a_n = (a_1)(r^{n-1})}$ Ex: Find the 5 term of a geometric sequence where a1 = 3, r = 2

$a_5 = (3)(2^{5-1})$
$= (3)(16)$→48

## Geometric Series

**Geometric Series-** The indicated sum of the terms of a geometric sequence
The **Formula for the Sum of a Geometric Series-** $\mathbf{S_n = \frac{a_1 - a_1r^n}{1-r}}$ **or** $\mathbf{S_n = \frac{a_1(1-r^n)}{1-r}}$ **or** $\mathbf{S_n = \frac{a_1 - a_nr}{1-r}}$

Ex: Find the sum of the geometric series where $a_1 = 5$ and $r = -2$

$Sn = \frac{a1 - a1rn}{1-r}$ → $Sn = \frac{5 - 5(-2)6}{1-(-2)}$
$= \frac{-315}{3}$ → -105

## Infinite Geometric Series

**Partial Sum-** In an infinite series, $S_n$ is called a partial sum because it is the sum of a certain number of terms and not the sum of the entire series
**Sum of an Infinite Geometric Series-** $\mathbf{S = \frac{a_1}{1-r}}$ iff $-1 < r < 1$ Ex: Find sum of infinite series if it exists: $1 - 3 + 9 - 27$ $a_1 = 1$ and $a_2 = -3$ so $r = \frac{-3}{1}$
Since $|-3| > 1$, no sum exists

## Recursion and Special Sequences

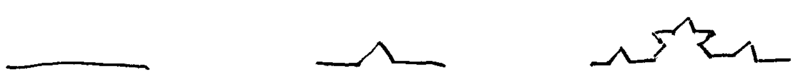
**Recursive Formula-** A recursive formula has the values of the first term(s), and a recursion equation that shows how to find each term from the term(s) before it.

| Sequence | Sequence Type | Recursive Formula |
|---|---|---|
| 9, 13, 17,... | Arithmetic | $a_{n+1} = a_n + 4$, $a_1 = 9$, $n \geq 1$ |
| 7, 21, 63,... | Geometric | An+1 = an(3), a1 = 7, $n \geq 1$ |
| 1, 1, 2, 3, 5, 8,... | Fibonacci | $a_n = a_{n-1} + a_{n-2}$, $a_1 = 1$, $a_2 = 1$, $n \geq 3$ |

Find the first three terms of the sequence where $a_1 = 5$ $a_{n+1} = 2a_n + 3$, $n \geq 1$
$a_1 = 5$
$a_{1+1} = 2a_1 + 3$
$a_2 = 2(5) + 3$ → 13
$a_{2+1} = 2a_2 + 3$
$a_3 = 2(13) + 3$ → 29
The first three terms are 3, 13, and 29

## Fractals

**Fractal-** A geometric figure that has self–similarity, is created using a recursive process, and is infinite in structure.
**Self-Similarity-** A characteristic of an object in which replicas of an entire shape or object are embedded over and over again inside the object in different sizes
Ex:



Stage 1 Stage 2 Stage 3

## The Binomial Theorem

**Pascal's Triangle-** The pyramid formation of the coefficients of binomial expansion
**Binomial Theorem-** $\mathbf{(a+b)^n = 1a^nb^0 + \frac{n}{1}(a^{n-1}b^1) + \frac{n(n-1)}{1*2}a^{n-2}b^2 + \ldots + 1a^0b^n}$

**Factorials-** If n is a positive integer, the expression n! is defined as $n! = n(n-1)(n-2)(n-3) \ldots 2 * 1$. By definition $0! = 1$ Ex: $4! = 4 * 3 * 2 * 1 = 24$
You can express the Binomial Theorem in sigma notation also→ $\frac{n!}{k!(n-k)!} a^{n-k} b^k$

Ex: Find the seventh term of $(p + q)^{11} = \frac{11!}{6!(11-6)!} p^{11-6} q^6$ → $(462)p^5q^6$