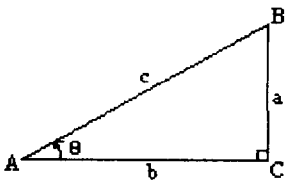
Trigonometric Functions

Basic Trigonometric Functions

trigonometry - the study of the relationships between the angles and sides of right triangles.



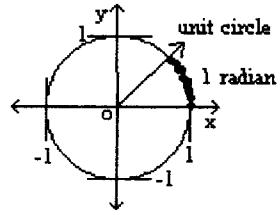
$\sin\theta = \frac{a}{c} or \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{b}{c} or \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{a}{b} or \frac{opposite}{adjacent}$

SOH-CAH-TOA Sine=Opposite over Hypotenuse, Cosine=Adjacent over Hypotenuse, Tangent=Opposite over Adjacent

$\csc\theta = \frac{c}{a} or \frac{hypotenuse}{opposite}$ $\sec\theta = \frac{c}{b} or \frac{hypotenuse}{adjacent}$ $\cot\theta = \frac{b}{a} or \frac{adjacent}{opposite}$ csc = cosecant, sec = secant, cot= cotangent

solving a triangle - finding the lengths of all the sides and the measures of all the angles
angle of elevation - the angle formed by the line of sight from a lower object and a horizontal line
angle of depression - the angle formed by the line of sight from a higher object and a horizontal line

Angles and Angle Measure

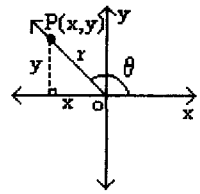


initial side - the side of the angle that is fixed on the positive x-axis
terminal side - the side of the angle that can rotate about the origin
standard position - when an angle's initial side is on the positive x-axis and the vertex is on the origin
radian - the measure of an angle that intercepts an arc whose length is one unit π radians=180°
unit circle - a circle with a radius of one
coterminal angles - angles that have the same terminal side

Trigonometric Functions of General Angles

For any angle in standard position with measure θ, a point P(x,y) on its terminal side, and $r = \sqrt{x^2 + y^2}$, trigonometric functions of θ are as follows.

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$ $\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$



For a 30-60-90 triangle (an angle with a 30°angle, a 60°angle, and a 90°angle), the ratio of the sides is $1 : \sqrt{3} : 2$ and for a 45-45-90 triangle, the ratio of the sides is $1 : 1 : \sqrt{2}$.

trigonometric identities - trigonometric expressions that hold true for any value of θ.

quotient identities - $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

reciprocal identities - $\sin\theta = \frac{1}{\csc\theta}$, $\csc\theta = \frac{1}{\sin\theta}$, $\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$, $\tan\theta = \frac{1}{\cot\theta}$, and $\cot = \frac{1}{\tan\theta}$

Law of Sines

(refer to the triangle at the top of the page)

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

(refer to the triangle at the top of the page)

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

when to use the law of sines	when to use the law of cosines
-two sides and an angles opposite one of the sides are known. This case has zero, one, or two solutions -two angles and a side are known	-two sides and the included angles are known -three sides are known

Circular Functions

If the terminal side of an angle θ in standard position intersects the unit circle at P(x,y), then $\cos\theta = x$ and $\sin\theta = y$.

periodic function - a function is called periodic if there is a number such that f(x) = f(x+a) for all x in the domain of the function. The least positive value of for a which f(x)=f(x+a) is called the period of the function.

Inverse Trigonometric Functions

principal values - the values in the restricted domains of the functions

y=Cos x iff y=cos x and $0 \le x \le \pi$ y=Sin x iff y=sin x and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ y=Tan x iff y=tan x and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Given y=Cos x, the inverse is $y=Cos^{-1}x$ or y=Arccos x
Given y=Sin x, the inverse is $y=Sin^{-1}x$ or y=Arcsin x
Given y=Tan x, the inverse is $y=Tan^{-1}x$ or y=Arctan x