Solving Systems of Linear Equations and Inequalities

Graphing Systems of Equations

system of equations - A set of equations with the same variables.

Graphs of Equations	Slopes and Intercepts	Name of System of Equations	Number of Solutions
lines intersect	different slopes	consistent and independent	one
lines coincide	same slope, same intercepts	consistent and dependent	infinite
lines parallel	same slope, different intercepts	inconsistent	zero

Solving Systems of Equations Algebraically

Ex. $\begin{cases} a+b=8 \\ b-2=a \end{cases}$

substitution method

(1) Solve the easier equation for one of the variables. $a+b=8 \rightarrow b=-a+8$

(2) Substitute the expression for the variable in the other equation. $(-a+8)-2=a \rightarrow a=3$

(3) Substitute the known variable into one of the equations and solve for the second variable. $3+b=8 \rightarrow b=5$

elimination method

(1) Compare the coefficients. $\begin{cases} a+b=8 \\ b-2=a \end{cases} \rightarrow \begin{cases} a+b=8 \\ a-b=-2 \end{cases}$

(2) Add or subtract one equation to or from the other equation.

$$\begin{array}{l} \quad a+b=8 \\ (+)\,a-b=-2 \\ \hline \quad 2a \quad =6 \quad \therefore \quad a=3 \end{array}$$

(3) Substitute the known variable into one of the equations and solve. $3+b=8 \rightarrow b=5$

Cramer's Rule

Gabriel Cramer was a Swiss mathematician who published the method of solving systems of equations using determinants.

The solution to the system $\begin{cases} ax+by=e \\ cx+dy=f \end{cases}$ is (x, y), where $x=\dfrac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y=\dfrac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, and \begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$. reminder: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$

Graphing Systems of Inequalities

Graph inequalities as lines first, and then test points on both sides of the line to know which region to shade.

Linear Programming

constraints - The inequalities whose graphs form the boundaries of the graph of the system's solution.
feasible region - The area of the intersection of the graphs in which every constraint is met.
linear programming - A method for finding the maximum or minimum value of a function subject to given constraints .
unbounded - When a polygonal region is not formed by the constraints.
Graph the constraints, and test points on both sides of the constraints to know which region to shade.

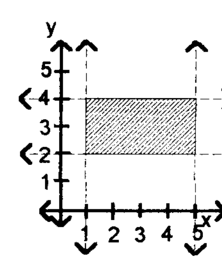
Applications of Linear Programming

linear programming procedure:

1. Define the variables.
2. Write a system of inequalities.
3. Graph the system of inequalities
4. Find the coordinates of the vertices of the feasible region.
5. Write an expression to be maximized or minimized.
6. Substitute the coordinates of the vertices into the expression.

Ex. Find the maximum and minimum profits if the function is $f(x.y)=2x+4y$ and the constraints are: $1<x<5$ and $2<y<4$

The vertices are at (1,2), (1,4), (5,2), (5,4).



(x,y)	2x+4y	f(x,y)
(1,2)	2(1)+4(2)	10
(1,4)	2(1)+4(4)	18
(5,2)	2(5)+4(2)	18
(5,4)	2(5)+4(4)	26

The maximum is (5,4) and the minimum is (1,2).

Solving Systems of Equations in Three Variables

ordered triple - The solution of a system of equations in three variables.
octants - Three mutually perpendicular planes separate space into eight regions, each called an octant.

three planes intersect at one point	three planes intersect in a line	three planes coincide	two planes coincide and intersect third in a line	three planes have no point in common
one unique solution	infinite # of solutions	infinite # of solutions	infinite # of solutions	no solutions