Exploring Polynomials and Radical Expressions

Monomials

Scientific Notation- A way of expressing extremely large or small numbers as $a \times 10^n$ where $1 \le a \le 10$ and n is an integer

Ex: $4 \times 10^8 = 400,000,000$

$3 \times 10^{-6} = .000003$

Monomial- an expression that is a number, variable, or product of a number and a variable. Ex: $4a^2b$

Constants- monomials with no variables

Coefficient-numerical factor of a monomial

Degree of a monomial- sum of all the monomials variables' exponents. Ex: $3x^2y^3$ 2+3=5 A 5 degree monomial

Power-an expression in the form of y^n

Simplify- rewrite an expression without parentheses or negative exponents

Exponent Rules

Negative Exponent Rule	If $a \neq 0$ $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$
Multiplying Powers	$a^m * a^n = a^{m+n}$
Dividing Powers	If $a \neq 0$ $\frac{a^m}{a^n}$ a^{m-n}
Power of a Power	$(a^m)^n = a^{m*n}$
Power of a Product	$(ab)^m = a^m * b^m$
Power of a Quotient	$(a/b)^n = a^n/b^n$ and $(a/b)^{-n} = (b/a)^n$ or b^n/a^n

Polynomials

Like terms- two monomials that are the same or differ only by their coefficients

Polynomial- a monomial or a sum of monomials

Terms- monomials that make up a polynomial

Binomial- a polynomial with 2 terms

Trinomial- a polynomial with 3 terms

Degree of a polynomial- The degree of the monomial with the highest degree

Foil Method of multiplying polynomails:

First Term
Outer Term
Inner Term
Last Term

Dividing Polynomials

Synthetic Division-a simpler method than long division used to divide a polynomial by a binomial

Ex: $(6x^3-19x^2+x+6)\div(x-3)$

Step 1: Write terms of polynomial so degrees are in descending order. Then write the coefficients as shown. $6 \quad -19 \quad 1 \quad 6$

Step 2: Write constant of r of the divisor x-r to the left. $3 \,|\, 6 \quad -19 \quad 1 \quad 6$

Step 3: Bring first coefficient down as shown 6

Step 4: Multiply first coefficient by r. Write product under second coefficient. Add product and second coefficient.

$$\begin{array}{r|rrrr} 3 & 6 & -19 & 1 & 6 \\ & & 18 & -3 & -6 \\ \hline & 6 & -1 & -2 & \boxed{0} \end{array}$$

Step 5: Multiply sum by r. Write product under next coefficient and add

Step 6: Repeat Step 5 for as many coefficients as there are. Then write the remainder.

Step 7: Rewrite equation with one degree less for each coefficient. $6x^2 - x - 2$

Factoring

Factor- a number, variable, monomial, or polynomial multiplied to obtain a product.

	Any Number of Terms
GCF	ab+ac+ad= a(b+c+d)
	Two Terms
Difference of Two Squares	$a^2-b^2=(a+b)(a-b)$
Sum of Two Cubes	$a^3+b^3=(a+b)(a^2-ab+b^2)$
Difference of Two Cubes	$a^3-b^3=(a-b)(a^2+ab+b^2)$
	Three Terms
Perfect Square Trinomials	$a^2+2ab+b^2=(a+b)^2$
	$a^2-2ab+b^2=(a-b)^2$
General Trinomials	$acx^2+(ad+bc)x+bd=(ax+b)(cx+d)$
	Four or More Terms
Grouping	ra+rb+sa+sb= r(a+b)+s(a+b)= (r+s)(a+b)

Factoring Steps

1. Find GCF
2. Look for a pattern
   (difference of 2 squares
   (perfect square trinomials
   (sum or difference of 2 cubes
3. If a polynomial is a square trinomial use British Method
4. If $\geq$ 4 terms try grouping

British Method

Look for 2 numbers whose product is ac and sum is b Ex:

Roots of Real Numbers

Square Root- For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
Christoff Rudolff- Introduced the radical sign in his algebra book *Die Cross*. He probably chose it because it resembled an r, the first letter in radix, which means root.
Definition of nth root- For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Principal Root- The nonnegative root. If there is a negative root but no nonnegative root, the principal root is the negative root.

Radical Expressions and Rational Exponents

Radical Properties

Product Property- $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ when a and b are both nonnegative and n>1
Quotient Property- if b≠0 and n>1 then $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
Definition of $b^{\frac{1}{n}}$- if n>1 then $b^{\frac{1}{n}} = \sqrt[n]{b}$ except when b<0 and n is even
Definition of Rational Exponents- If b is nonzero and n>1 then $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$ except when b<0 and n is even

Ex: Simplify $7\sqrt[3]{27n^2} * 4\sqrt[3]{8n}$

$7\sqrt[3]{27n^2} * 4\sqrt[3]{8n} = 7 * 4 * \sqrt[3]{27n^2 * 8n}$ → product property
$= 28 * \sqrt[3]{3^3 * 2^3 * n^3}$ → factor into cubes where possible
$= 28 * \sqrt[3]{3^3} * \sqrt[3]{2^3} * \sqrt[3]{n^3}$ → product property
$= 28 * 3 * 2 * n$ → 168n

Ex: Simplify $\sqrt[4]{4n^2}$

$\sqrt[4]{4n^2} = (4n^2)^{(1/4)}$ → Rational Exponents
$(2^2 * n^2)^{(1/4)}$ →Simplify
$2^{2(1/4)} * n^{2(1/4)}$ →Distribute
$2^{(1/2)} * n^{(1/2)}$ →Simplify
$\sqrt{2} * \sqrt{n}$ → $\sqrt{2n}$

Solving Radical Equations and Inequalities

Radical Equations- Equations that contain radical expressions with variables in the radicand
Extraneous Solutions- Solutions that do not satisfy the original equation

Ex: Solve $2(7n - 1)^{(1/3)} - 4 = 0$
$2(7n - 1)^{(1/3)} - 4 = 0$
$2(7n - 1)^{(1/3)} = 4$ → add 4 to each side
$(7n - 1)^{(1/3)} = 2$ → divide by 2 to isolate the cube root
$[(7n - 1)^{(1/3)}]^3 = 2^3$ → cube each side to get rid of rational exponent
$7n - 1 = 8$
$7n = 9$ → $n = (9/7)$ Now check the equation for extraneous solutions

Check: $2(7n - 1)^{(1/3)} - 4 = 0$
$2[7(9/7) - 1]^{(1/3)} - 4 = 0$
$2(8)^{(1/3)} - 4 = 0$
$2(2) - 4 = 0$
$0 = 0$ ✓
The solution is (9/7)

Solving Radical Inequalities
1. Solve the equation in the radical expression first
2. Solve the whole radical equation.
3. Check to see if the equation makes sense with a drawing on the number line.

Complex Numbers

Pure imaginary numbers- A complex number of the form **bi,** where b is real and **i** is the imaginary unit.
Definition of Pure Imaginary Numbers- $\sqrt{-b2} = \sqrt{b2} * \sqrt{-1}$ or bi
Complex Number- Any number that can be written in the form a + **b**i, where a and b are real numbers and **i** is the imaginary unit.

Ex: Simplify $\sqrt{-12}$
$\sqrt{-12} = \sqrt{2^2} * \sqrt{3} * \sqrt{-1}$
$= 2 * \sqrt{3} * i$
$= 2i\sqrt{3}$

Simplifying Expressions Containing Complex Numbers

Complex Conjugates- Complex numbers of the form a + **b**i and a – **b**i

Ex: Simplify $\frac{8i}{1 + 3i}$

$\frac{8i}{1+3i} = \frac{8i}{1+3i} \cdot \frac{1-3i}{1-3i}$ ← multiply by its complex conjugate to simplify

$= \frac{8i(1-3i)}{1+3^2}$

$= \frac{8i-24i^2}{10}$

$= \frac{8i+24}{10}$

$= \frac{12+4i}{5}$