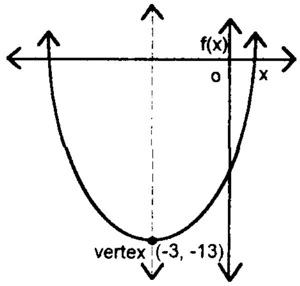
Quadratic Functions and Inequalities

Solving Quadratic Equations by Graphing

quadratic function - A function described by an equation that can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.

leading co-efficient - a **quadratic term** - ax^2 **linear term** - bx **constant term** - c

parabola - the graph of any quadratic function, and the loci of points equidistant from a point (the focus) and a line (the directrix)



axis of symmetry - the line that splits the graph symmetrically in half
vertex - the point where the parabola intersects the axis of symmetry
zeros - the x-coordinates of the points where the parabola intersects the x-axis

If $a > 0$, then the parabola opens up.
If $a < 0$, then the parabola opens down.

quadratic equation - when a quadratic function is set equal to zero

axis of symmetry: $x = -3$

roots - solutions of a quadratic equation

Solving Quadratic Equations by Factoring

factoring - solving a quadratic equation by using the zero product property

zero product property - For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both.

Ex. $0 = 48t - 16t^2 \quad \rightarrow \quad 0 = 16t(3 - t)$

Because $16t$ times $3 - t$ is 0, either $16t$ or $3 - t$ equals zero. $16t = 0$ or $3 - t = 0$

$t = 0 \qquad t = 3$

Completing the Square

Ex. $x^2 - 6x = 40$

1. Find the term that completes the square on the left side. $\quad x^2 - 6x + \square = 40 + \square$
   *take one half of b and square it
2. Add the term to both sides $\quad x^2 - 6x + 9 = 40 + 9$
3. Factor $\quad (x-3)^2 = 49$
4. Take the square root of each side $\quad x - 3 = \pm 7$
5. Solve for x and substitute to check. $\quad x = 10$ and $x = -4$

The Quadratic Formula and the Discriminant

The quadratic formula - The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant - $b^2 - 4ac$

Value of $b^2 - 4ac$	Discriminant a perfect square?	Nature of Roots	Nature of related graph
$b^2 - 4ac > 0$	yes	2 real, rational	intersects x-axis twice
$b^2 - 4ac > 0$	no	2 real, irrational	intersects x-axis twice
$b^2 - 4ac < 0$	--	2 imaginary	does not intersect x-axis
$b^2 - 4ac = 0$	--	1 real	intersects x-axis once

Sum and Product of Roots

If the roots of $ax^2 + bx + c = 0$ with $a \neq 0$ are s_1 and s_2, then $s_1 + s_2 = -\frac{b}{a}$ and $s_1 \cdot s_2 = \frac{c}{a}$.

Analyzing Graphs of Quadratic Functions

hk form - $y = a(x-h)^2 + k$, where (h, k) is the vertex and the axis of symmetry is at $x = h$.

a determines which way the graph opens. a is positive $\rightarrow$ graph opens upward. a is negative $\rightarrow$ graph opens downward.

As the value of $|a|$ increases, the graph narrows.

Graphing and Solving Quadratic Inequalities

boundary - a line or curve that separates a graph into two parts

To graph a quadratic inequality:
1) Graph the boundary. Determine if it should be solid or dashed ($\leq$ or $\geq$ $\rightarrow$ use a solid line, $<$ or $>$ $\rightarrow$ use a dashed line)
2) Test a point in each region
3) Shade the region whose ordered pair results in a true inequality

Standard Deviation

standard deviation - a measure of variation that measures how much each value in a set of data differs from the mean.

To find the standard deviation of a set of data:

1) Find the mean, $\bar{x}$
2) Find the difference between each value in the set and the mean
3) Square each difference
4) Find the mean of the squares
5) Take the principal square root of this mean

N	X	$(x_n - \bar{x})$	$(x_n - \bar{x})^2$
1	x_1	$x_1 - \bar{x}$	$(x_1 - \bar{x})^2$
2	x_2	$x_2 - \bar{x}$	$(x_2 - \bar{x})^2$
3	x_3	$x_3 - \bar{x}$	$(x_3 - \bar{x})^2$
4	x_4	$x_4 - \bar{x}$	$(x_4 - \bar{x})^2$
⋮	⋮	⋮	⋮

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

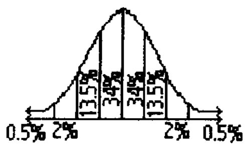
$$\overline{(x_n - \bar{x})^2} = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$

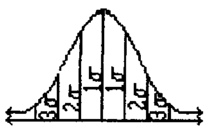
$$\text{SD} = \sigma = \sqrt{\overline{(x_n - \bar{x})^2}}$$

Approx. 68% of data will be within $\pm 1\sigma$ of $\bar{x}$

Approx. 95% of data will be within $\pm 2\sigma$ of $\bar{x}$

Approx. 99% of data will be within $\pm 3\sigma$ of $\bar{x}$





The Normal Distribution

frequency distribution - shows how data are spread out over the range of values
histogram - a bar graph that displays a frequency distribution
normal distribution - a data distribution that gives a bell-shaped, symmetrical graph
bell curve - a symmetric curve that is the general shape of the graph of a normal distribution
skewed distribution - a distribution curve that is not symmetric

The bell curve is also known as the Gaussian curve, because Gauss discovered the curve.