Exploring Polynomial Functions

Polynomial Functions

Polynomial in one variable- a polynomial of degree n in one variable x is an expression of the from $a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$ where the coefficients $a_0, a_1, a_2, \ldots a_n$ represent real numbers, a_0 is not zero and n is a nonnegative integer Ex: $6x^2 + 4x$ – yes, only x

$9x^3 + 4y$ – no, two variables

$t^{-3} + 2t$ – no, negative exponent

$3x^2 + (2/x)$—no, can't be written x^n

Substituting a polynomial in 1 variable

Find p(a+2) if $p(x) = x^3 - 2x + 1$

$p(a+2) = (a+2)^3 - 2(a+2) + 1$

$= a^3 + 6a^2 + 12 + 8 - 2a - 4 + 1$

$= a^3 + 6a^2 + 10a + 5$

The Remainder and Factor Theorems

Remainder Theorem- If polynomial f(x) is divided by x-a, the remainder is the constant f(a), and the dividend= quotient*divisor+remainder where q(x) is a polynomial with degree one less than the degree of f(x)

Ex: $f(x)= 3x^4 - x^3 + 2x - 6$ Show that f(2) is the remainder when f(x) is divided by x-2

Factor Theorem- The binomial (x-a) is a factor of the polynomial f(x) iff f(a)=0

Synthetic Substitution- using synthetic division to find the value of a function

Depressed Polynomial- The quotient of dividing a polynomial by one of its factors

$$\begin{array}{r|rrrrr} 2 & 3 & -1 & 0 & 2 & -6 \\ & & 6 & 10 & 20 & 44 \\ \hline & 3 & 5 & 10 & 22 & 38 \end{array}$$

$f(2) = 3(2)^4 - 2^3 + 2(2) - 6$

$= 48 - 8 + 4 - 6$

$= 38$ ✓

Graphing Polynomial Functions and Approximating Zeros

Location Principle- If y=f(x) and a and b are 2 numbers such that f(a) is negative and f(b) is positive then the function has at least 1 real zero between a and b.

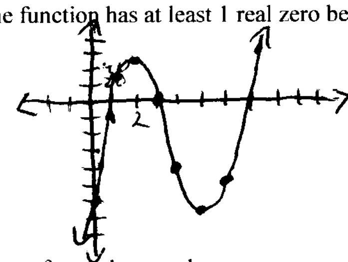
Example: $V(t)=9t^3 - 93t^2 + 238t - 120$

t	V(t)
0	-120
1	34
2	56
3	0
4	-80
5	-130
6	-96
7	76

zero between t=0 and t=1

zero at t=3

zero between t=6 and t=7



Roots and Zeros

Fundamental Theorem of Algebra- an Nth degree polynomial equation will have N terms

Corollary- A polynomial equation P(x)=0 of degree N with complex coefficients has exactly N roots in the set of complex numbers

Complex Conjugates Theorem- a and b are real numbers with b=0. If a+bi is a zero of a polynomial function, then a-bi is also a zero.

Ex: If 4-i and (–3) are zeros of a 3rd degree equation then what is the other zero? 4+i because 4+i is the complex conjugate of 4-i

Descartes' Rule of Signs- If P(x) is a polynomial function whose terms are arranged in descending powers of the variable: The number of positive real zeros of P(x) is the same as the number of changes in sign of the coefficients of the terms, or is than this by an even number, and the number of negative real zeros of P(x) is the same as the number of changes in sign of the coefficients of the terms of P(-x), or is less than this by an even number.

	Zeros		
+ zeros	4	2	0
- zeros	1	1	1
i zeros	0	2	4

Ex: State the number of positive and negative real zeros for p(x)=4x5 + 3x4 – 2x3 + 5x2 – 6x + 1.

p(x)=4x5 + 3x4 – 2x3 + 5x2 – 6x + 1.

4 3 -2 5 -6 1

no yes yes yes yes → four sign changes 4, 2, or 0 positive real zeros

p(-x)=-4x5 + 3x4 + 2x3 + 5x2 + 6x + 1

-4 3 2 5 6 1

yes no no no no → 1 sign change 1 negative real zero

Rational Zero Theorem

Rational Zero Theorem- Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots + a_{n-2}x^2 + a_{n-1}x + a_n$ represent a polynomial function with integral coeffiecients. If p/q is a rational number in simplest form and is a zero of y = f(x), then p is a factor of a_n and q is a factor of a_0

Corallary- If the coefficients of a polynomial function are integers such that $a_0 = 1$ and $a_n = 0$, any rational zeros of the function must be factors of a_n.

Mneumonic for Rational Zero Theorem → The denominator of a root is a factor of the leading coefficient. The numerator is a factor of a constant.

Ex: Find all possible rational zeros of $f(x)= 3x^3 + 9x^2 + x - 10$ p/q is a rational root so p is a factor of –10 and q is a factor of 3 (remember mneumonic) Possible values of p are 10, 5, 2, 1. Possible values of q are 1 and 3. So all possible rational zeros are 1, 2, 5, 10, 1/3, 5/3, 10/3. Use Descartes Rule of Signs to find out if values can be negative, positive, or both.

Using Quadratic Techniques to Solve Polynomial Equations

In order to solve some polynomial equations you need to use quadratic form to solve it in an easier way.

Quadratic Form- For any numbers a, b, and c, except a=0, $a[f(x)]^2 + b[f(x)] + c = 0$.

Ex: $x^4 - 17x^2 + 16 = 0$

$(x^2)^2 - 17(x^2) + 16 = 0$. → Quadratic Form

$(x^2 - 16)(x^2 - 1) = 0$

$(x - 4)(x + 4)(x - 1)(x + 1) = 0$. Therefore roots are –4, 4, -1, and 1 using the zero product property

Composition of Functions

Composition of Functions- Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composite function f g can be described by the equation [f g](x) = f[g(x)] which is read "f of g of x"

To find [f g](x) you must find g(x) and then find f(that answer) Ex: Find [f g](x) if $f(x) = x^2 - 4$ and $g(x) = 4x - 1$

Subsitute 4x – 1 for g(x) f(4x – 1) since g(x) = 4x – 1

$(4x - 1)^2 - 4$

$16x^2 - 8x + 1 - 4 \rightarrow 16x^2 - 8x - 3$

Inverse Functions and Relations

Inverse Functions- Two functions f and g are inverse functions iff both of their compositions are the identity function. That is, [f g](x) = x and [g f](x) = x.

Property of Inverse Functions- Suppose f and f^{-1} are inverse functions. Then f(a) = b iff $f^{-1}(b) = a$.

Definition of Inverse Relations- Whenever one relation contains the element (a,b), the other relation contains the element (b,a).

To find the inverse function you must switch the x and y in the equation and solve for y. Ex: Find the inverse of f(x) = 3x + 6 → y = 3x + 6

Switch x and y x = 3y + 6

Solve for y y = x – 6