Rational Expressions

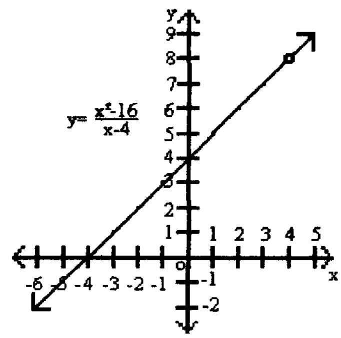
Graphing Rational Functions

rational function - a function that is the ratio of two polynomial expressions. $f(x) = \dfrac{p(x)}{q(x)}$ p(x) and q(x) are polynomial functions. $q(x) \neq 0$

continuity - can be traced with a pencil that never leaves the paper

point discontinuity - a hole in the graph - a break in continuity **Ex.** $\dfrac{x^2-16}{x-4}$

x	y
-6	-2
-5	-1
-4	0
-3	1
-2	2
-1	3
0	4
1	5
2	6
3	7
4	undef
5	8



Direct, Inverse, and Joint Variation

direct variation - y varies directly as x if there is some nonzero constant k such that $y = kx$, k is called the constant of variation. As x increases, y increases and decreases at a constant rate.

inverse variation - y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \dfrac{k}{x}$.

joint variation - y varies jointly as x and z if there is some number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.

Multiplying and Dividing Rational Expressions

rational algebraic expression - an algebraic expression that can be expressed as the quotient of two polynomials where the denominator does not equal zero.

For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\dfrac{a}{b}\cdot\dfrac{c}{d} = \dfrac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$, and $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b}\cdot\dfrac{d}{c} = \dfrac{ad}{bc}$, if $b \neq 0, c \neq 0$, and $d \neq 0$.

Ex. Simplify $\dfrac{8x^2y}{a^3b} \div \dfrac{4x^3y^2}{a^2b^3}$.

$$\frac{8x^2y}{a^3b} \div \frac{4x^3y^2}{a^2b^3} = \frac{8x^2y}{a^3b}\cdot\frac{a^2b^3}{4x^3y^2} = \frac{8^2 x^{2^0} y}{a^{3}{}_1 b}\cdot\frac{a^{2^0} b^{3^2}}{4_1 x^{3}{}_1 y^{2}{}_1} = \frac{2b^2}{axy}$$

complex fraction - a rational expression whose numerator and/or denominator contains a rational expression. **Ex.** $\dfrac{\frac{xy}{z}+4}{6x-5y}$ is a complex fraction.

Adding and Subtracting Rational Expressions

$$\frac{a}{c}+\frac{b}{d} = \frac{a\cdot d}{c\cdot d}+\frac{b\cdot c}{c\cdot d} = \frac{ad+bc}{cd}$$

$$\frac{a}{c}+\frac{b}{d} = \frac{a\cdot d}{c\cdot d}+\frac{b\cdot c}{c\cdot d} = \frac{ad+bc}{cd}$$

Ex. $\dfrac{3x^2}{2y} \div \dfrac{3y}{5x} = \dfrac{3x^2}{2y}\cdot\dfrac{5x}{3y} = \dfrac{15x^3}{6y^2} = \dfrac{5x^3}{2y^2}$

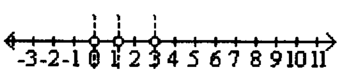
Solving Rational Equations and Inequalities

rational equation - an equation that contains one or more rational expressions

Ex.

$$\frac{x-2}{x} < \frac{x}{x-3} \rightarrow \frac{x-2}{x} - \frac{x}{x-3} < 0 \rightarrow \frac{(x-2)(x-3)-x(x)}{x(x-3)} < 0 \rightarrow \frac{x^2-5x+6-x^2}{x(x-3)} < 0 \rightarrow \frac{-5x+6}{x(x-3)} < 0$$

$x \neq 0, 3, \dfrac{6}{5}$



Try x= -1	Try x=1	Try x=2	Try x=4
$\dfrac{-1-2}{-1} < \dfrac{-1}{-1-3}$	$\dfrac{1-2}{1} < \dfrac{1}{1-3}$	$\dfrac{2-2}{2} < \dfrac{2}{2-3}$	$\dfrac{4-2}{4} < \dfrac{4}{4-3}$
$\dfrac{-3}{-1} \not< \dfrac{-1}{-4}$	$\dfrac{-1}{1} < \dfrac{1}{-2}$	$\dfrac{0}{2} \not< \dfrac{2}{-1}$	$\dfrac{2}{4} < \dfrac{4}{1}$

$0 < x < \dfrac{6}{5}$ and $x > 3$.