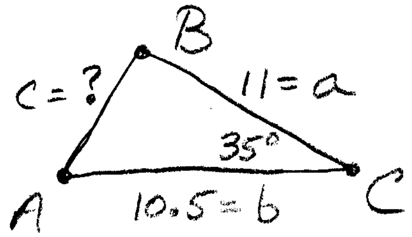
BE - Alg 2 WEDNESDAY 1-20-10

* Homework review
Pg 736 # 4, 11, 17

Homework Review – Pg 736 # 4, 11, 17

4.



SAS ⇒ 2 SIDES ⇒ L of C

Find c first, sides to nearest tenth, angles to nearest degree.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 11^2 + 10.5^2 - 2(11)(10.5)\cos 35^\circ$$

$$c^2 = 121 + 110.25 - 231(.8192)$$

$$c^2 = 231.25 - 189.24$$

$$c^2 = 42.01$$

$$c = \sqrt{42.01} \cong 6.48 = \boxed{6.5 = c}$$

Find A ⇒ $\dfrac{\sin A}{11} = \dfrac{\sin 35^\circ}{6.48}$ $\therefore$ $\sin A = 11\left(\dfrac{\sin 35^\circ}{6.48}\right)$

DO NOT USE THE ROUNDED 6.5!

$$\sin A = 11\left(\frac{.5736}{6.48}\right)$$

$$\sin A = 11(.0885)$$

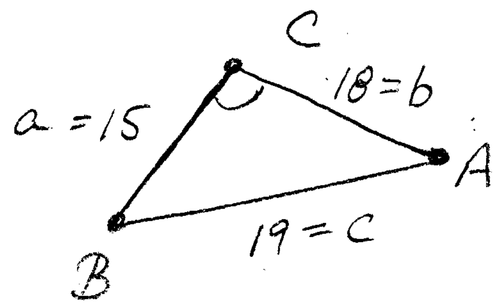
$$\sin A = .9735$$

$$A = \sin^{-1}(.9735)$$

$$A = 76.8 \quad \boxed{77^\circ\ A}$$

$\therefore$ $B = 180 - 77 - 35 = \boxed{68^\circ = B}$

11.



LOOK

* Tip: find biggest angle ⇒ angle C first ⇒

$$c^2 = a^2 + b^2 - 2ab\cos C$$ Get "By itself"

$$19^2 = 15^2 + 18^2 - 2(15)(18)\cos C$$

$$361 = 225 + 324 - 540\cos C$$

$$361 = 549 - 540\cos C$$

$$-549 \quad -549$$

$$\frac{-188}{-540} = \frac{-540\cos C}{-540}$$

$.3481 = \cos C$ ∴ $C = \cos^{-1}(.3481) = 69.6°$

∴ $\boxed{C = 70°}$

Find middle side's opposite angle B ⇒

$\frac{\sin B}{18} = \frac{\sin 70}{19}$ ∴ $\sin B = \frac{18\sin 70}{19}$

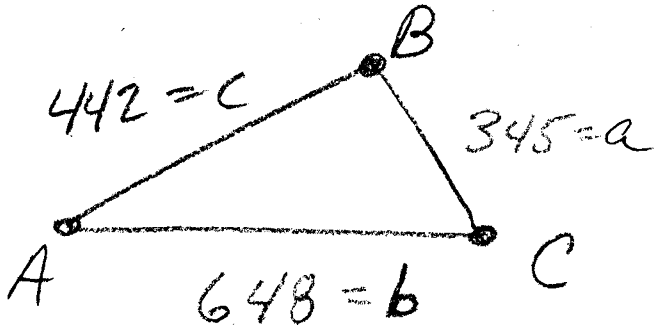
$$\sin B = \frac{18(.9397)}{19} = .8902$$

$\sin^{-1}(.8902) = \boxed{62.9° = 63° = B}$

∴ $C = 180 - 69.6 - 62.9$

$\boxed{C = 47.5 = 48°}$

(17) $a = 345$
$b = 648$
$c = 442$



LOOK * Find biggest side first

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$648^2 = 345^2 + 442^2 - 2(345)(442)\cos B$$

$$419904 = 119025 + 195364 - 304980 \cos B$$

$$419904 = 314389 - 304980 \cos B$$

$$105515 = -304980 \cos B$$

$$-0.3460 = \cos B \quad \therefore \cos^{-1}(-.3460) = 110.2^\circ$$

$$= \boxed{110^\circ = B}$$

Find middle side's opposite Angle $\Rightarrow C$

$$\frac{\sin C}{442} = \frac{\sin 110}{648} \quad \therefore \sin C = \frac{442(\sin 110)}{648}$$

$$= .6821(.9397)$$

$$\sin C = .6410$$

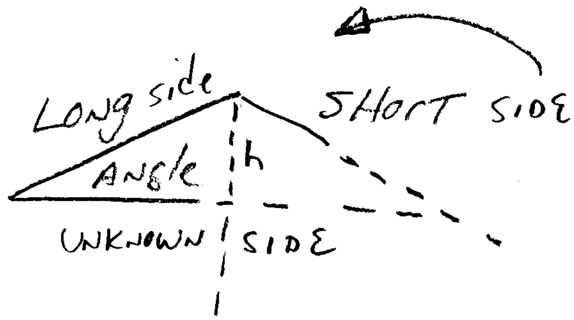
$$C = \sin^{-1}(.6410)$$

$$C = 39.9^\circ = \boxed{40^\circ\ C}$$

$$A \Rightarrow 180 - 110.2 - 39.9 = 29.2$$

$$\boxed{A = 30^\circ}$$

If you know two sides And one Angle, And the Angle is opposite the short side



SSA

There may be NONE, ONE, or two possible solutions.

The Law of Cosines results in A Quadratic with Ø positive, 1 positive, or 2 positive solutions.

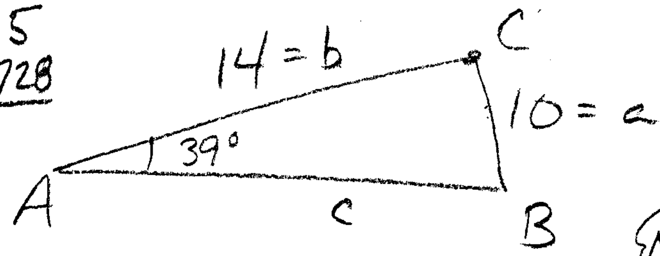
Recommended!

(see 727 & 728 if you want to use the Law of Sines)

* called the Ambiguous case of the LOS

KNOWN ANGLE OPPOSITE SHORT SIDE

EX 5
PG 728



SSA

TWO SIDES ⇒ LOC

NEED COS A FORM

LOC ⇒ KNOW a, b, COS A ONLY C IS UNKNOWN

⇒ "3 of 4" $\left\{ a^2 = b^2 + c^2 - 2bc\cos A \right.$

$$10^2 = 14^2 + c^2 - 2(14)c(\cos 39)$$

$$0 = 96 + c^2 - 28(.7771)c$$

$$c^2 - 21.76c + 96 = 0$$

$a = 1$ $\qquad b^2 - 4ac$

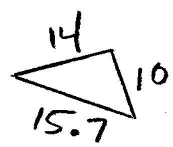
$b = -21.76$ $\qquad (-21.76)^2 - 4(1)(96)$

$c = 96$

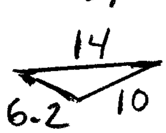
$$473.5 - 384 = 89.5 = d$$

$$c = \frac{-b \pm \sqrt{d}}{2a} = \frac{21.76 \pm \sqrt{89.5}}{2(1)}$$

$$c = \frac{21.76 \pm 9.46}{2}$$

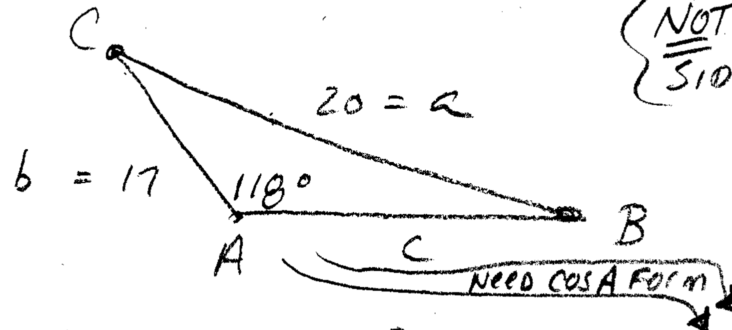


$$\boxed{c \cong \{15.7,\ 6.2\}}$$



TWO SOLUTIONS

Ex 3
Pg 727



{ KNOWN ANGLE IS NOT OPPOSITE SHORT SIDE – ONLY 1 SOLUTION }

{ SSA }

{ Two Sides LOC }

$$LOC \Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$$

$$20^2 = 17^2 + c^2 - 2(17)c\,(\cos 118^\circ)$$

$$400 = 289 + c^2 - 34(-.4695)c$$

$$0 = -111 + c^2 + 15.963c$$

OR $c^2 + 15.963c - 111 = 0$

$a = 1$ $\qquad b^2 - 4ac$

$b = 15.963$ $\qquad (15.963)^2 - 4(1)(-111)$

$c = -111$

$$254.82 + 444 = 698.82 = d$$

$$c = \frac{-b \pm \sqrt{d}}{2a} = \frac{-15.963 \pm \sqrt{698.82}}{2(1)}$$

$$c = \frac{-15.963 \pm 26.44}{2}$$

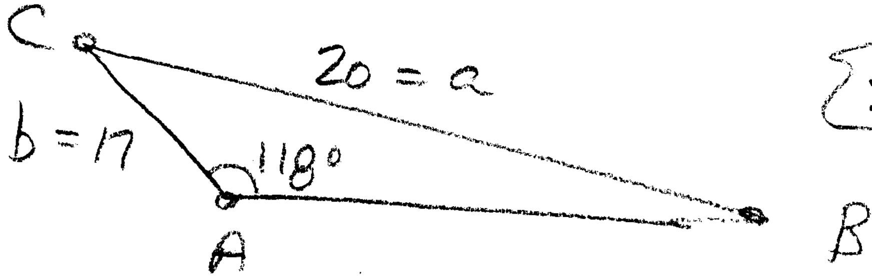
$c = 5.23$ AND -21.2 discard

$$\boxed{c = 5.23}$$

ONE SOLUTION

Ex 3
PG 727

Alternate solution using LoS
Since known angle is NOT opposite short side



SSA

$$\frac{\sin 118}{20} = \frac{\sin B}{17} \quad \therefore \sin B = 17\left(\frac{\sin 118°}{20}\right)$$

$$\sin B = 17\left(\frac{.8829}{20}\right)$$

$$\sin B = 17(.0441)$$

$$\sin B = .7497$$

$$B = \sin^{-1}(.7497)$$

$$\boxed{B = 48.6°}$$

$$\therefore C = 180 - 118 - 48.6 = \boxed{13.4° = C}$$

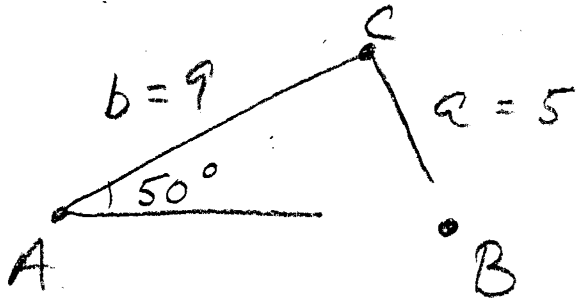
$$\therefore \frac{\sin 118}{20} = \frac{\sin 13.4}{c} \quad \therefore c = \sin 13.4\left(\frac{20}{\sin 118}\right)$$

$$c = .2317\left(\frac{20}{.8829}\right)$$

$$c = .2317(22.65)$$

$$\boxed{c = 5.25} \checkmark$$

EX 4
PG 728



KNOWN Angle opposite SHORT SIDE

$\boxed{SSA}$

2, 1, or 0 solutions

Know b, a, $\cos 50 = \cos A$ → Need $\cos A$ Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$5^2 = 9^2 + c^2 - 2(9)c(\cos 50)$$

$$0 = 56 + c^2 - 18(.6428)c$$

$$0 = c^2 - 11.57c + 56$$

$a = 1$
$b = -11.57$
$c = 56$

$$b^2 - 4ac$$

$$(-11.57)^2 - 4(1)(56)$$

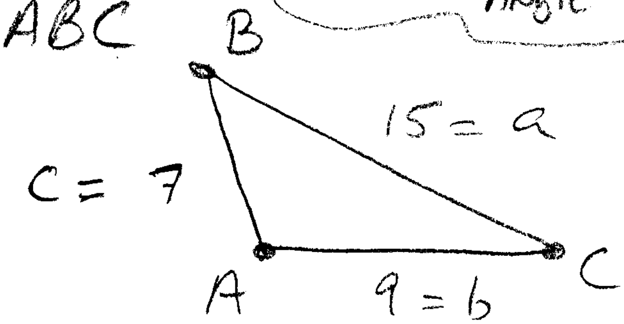
$$133.87 - 224 = -90.13 = d$$

d is negative

$\therefore$ $\boxed{\text{No Real Solutions}}$

SSS * Solve for Largest Angle 1st!!!

Ex 2 PG 734 Solve $\triangle ABC$



WHAT IF YOU DON'T?

SSS

Pick any of the 3 Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$7^2 = 15^2 + 9^2 - 2(15)(9)\cos C$

$49 = 225 + 81 - 270\cos C$

$49 = 306 - 270\cos C$

$-306 \quad -306$

$\dfrac{-257}{-270} = \dfrac{-270\cos C}{-270}$

$.9519 = \cos C \quad \therefore \quad C = \cos^{-1}(.9519) \doteq \boxed{17.8^\circ = C}$

Use LOS to find A or B (Don't use SSA short side) (TIP: Stay away from $A=7$)

$\dfrac{\sin A}{15} = \dfrac{\sin 17.8^\circ}{7} \quad \therefore \quad \sin A = 15\left(\dfrac{\sin 17.8}{7}\right)$

$\sin A = 15\left(\dfrac{.3057}{7}\right)$

* UH OH Ambiguous Case!!

$\sin A = .6551$

* $\therefore A = \sin^{-1}(.6551) = \boxed{40.9^\circ = A}$ *

$\therefore B = 180 - 17.8 - 40.9 = \boxed{121.3^\circ = B}$

!! Book $\Rightarrow A = 139^\circ$, $B = 23^\circ$!!

Summary: ASA, AAS } LAW OF SINES

SAS } LAW of COSINES

SSS } LAW of COSINES, solve for largest angle first

*SSA } LAW of COSINES ⇒ QUADRATIC EQUATION w/ 0, 1, 2 solutions.

*(IF KNOW ANGLE IS OPPOSITE SHORT SIDE)

- Homework: Pg 736 # 12, 13, 14

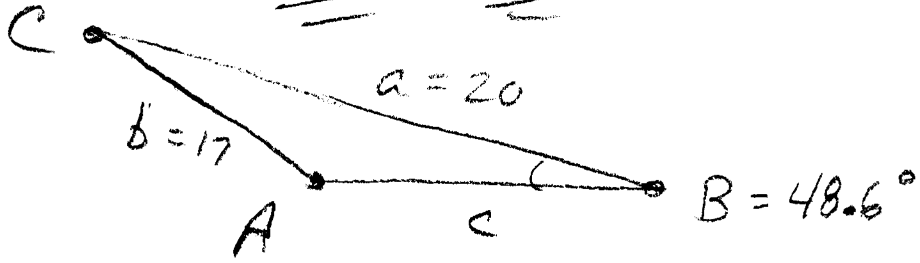
USE: LofS LofC LofC

QUADRATIC, SOLVE BOTH Δ

ONE MORE EXAMPLE

Pg 727

Lets try an "ambiguous" case of Ex. 3, lets put the known angle opposite the short side $b = 17$. $\Rightarrow$ 2, 1, or 0 SOLUTIONS



SSA

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$17^2 = 20^2 + c^2 - 2(20)(c)\cos 48.6^0$$

$$289 = 400 + c^2 - 40(.6613)c$$

$$0 = 111 + c^2 - 26.45c$$

$$\therefore\ c^2 - 26.45c + 111 = 0$$

$a = 1$

$b = -26.45$

$c = 111$

$$b^2 - 4ac$$

$$(-26.45)^2 - 4(1)(111)$$

$$699.60 - 444 = 255.6 = d$$

$$\text{"}x\text{"} = c = \frac{-b \pm \sqrt{d}}{2a} = \frac{26.45 \pm \sqrt{255.6}}{2(1)}$$

side c

$$= \frac{26.45 \pm 15.99}{2}$$

Now you have Two possible $\triangle$'s $\Rightarrow$ $c = \{21.22, 5.23\}$

(Since known angle is opposite short side)

LOOK {EX3 Pg 727}