$\boxed{\text{BE - Alg. 2}}$ WEDNESDAY 3-24-10

ACT
"REAL ACT"

① For all positive integers X, Y, Z, which of the following is Equivalent to $\frac{X}{Y}$?

Ⓐ $\frac{X \cdot Z}{Y \cdot Z}$

Ⓑ $\frac{X \cdot X}{Y \cdot Y}$

Ⓒ $\frac{Y \cdot X}{X \cdot Y}$

Ⓓ $\frac{X - Z}{Y - Z}$

Ⓔ $\frac{X + Z}{Y + Z}$

- Homework review — Pg. 445 # 4, 6, 7, 9

8-5 Hyperbolas

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Homework review
Pg. 445 # 4, 6, 7, 9

4. The center is the midpoint of the segment connecting the vertices, or (0, 0). The value of a is the distance from the center to a vertex, or 2 units. The value of c is the distance from the center to a focus, or 5 units. Use the equation $c^2 = a^2 + b^2$ to determine the value of b.

$c^2 = a^2 + b^2$
$5^2 = 2^2 + b^2$
$25 = 4 + b^2$
$21 = b^2$

Since the transverse axis is vertical, the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{y^2}{4} - \frac{x^2}{21} = 1$.

6. The equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, where $a = 3\sqrt{2}$ and $b = 2\sqrt{5}$. The vertices are at $(0, \pm 3\sqrt{2})$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

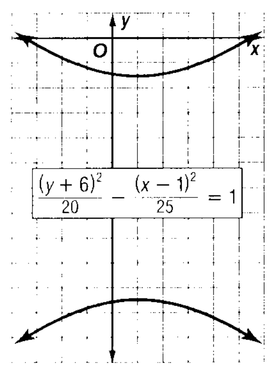
$c^2 = a^2 + b^2$
$c^2 = 18 + 20$
$c^2 = 38$
$c = \sqrt{38}$

Since $c = \sqrt{38}$, the foci are at $(0, \pm\sqrt{38})$. The equations of the asymptotes are $y = \pm \frac{3\sqrt{2}}{2\sqrt{5}}x$ or $y = \pm \frac{3\sqrt{10}}{10}x$.

7. The equation is of the form $\frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1$, where $h = 1$, $k = -6$, $a = 2\sqrt{5}$, and $b = 5$. The coordinates of the vertices are $(h, k \pm a)$ or $(1, -6 \pm 2\sqrt{5})$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 20 + 25$
$c^2 = 45$
$c = 3\sqrt{5}$

Since $c = 3\sqrt{5}$, the foci are at $(h, k \pm c)$ or $(1, -6 \pm 3\sqrt{5})$. The equations of the asymptotes are $y - (-6) = \pm \frac{5}{2\sqrt{5}}(x - 1)$ or $y + 6 = \pm \frac{2\sqrt{5}}{5}(x - 1)$.



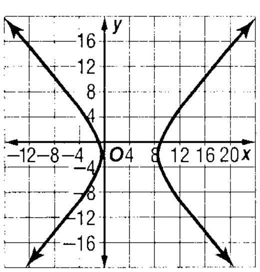
9. $$5x^2 - 4y^2 - 40x - 16y - 36 = 0$$
$$5(x^2 - 8x + \quad) - 4(y^2 + 4y + \quad) = 36 + 5(\quad) - 4(\quad)$$
$$5(x^2 - 8x + 16) - 4(y^2 + 4y + 4) = 36 + 5(16) - 4(4)$$
$$5(x - 4)^2 - 4(y + 2)^2 = 100$$
$$\frac{5(x - 4)^2}{100} - \frac{4(y + 2)^2}{100} = 1$$
$$\frac{(x - 4)^2}{20} - \frac{(y + 2)^2}{25} = 1$$

The equation is of the form $\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$, where $h = 4$, $k = -2$, $a = 2\sqrt{5}$, and $b = 5$. The coordinates of the vertices are $(h \pm a, k)$ or $(4 \pm 2\sqrt{5}, -2)$. Use the equation $c^2 = a^2 + b^2$ to determine the value of c.

$c^2 = a^2 + b^2$
$c^2 = 20 + 25$
$c^2 = 45$
$c = 3\sqrt{5}$

Since $c = 3\sqrt{5}$, the foci are located at $(h \pm c, k)$ or $(4 \pm 3\sqrt{5}, -2)$. The equations of the asymptotes are $y - (-2) = \pm \frac{5}{2\sqrt{5}}(x - 4)$ or

$y + 2 = \pm \frac{\sqrt{5}}{2}(x - 4)$.

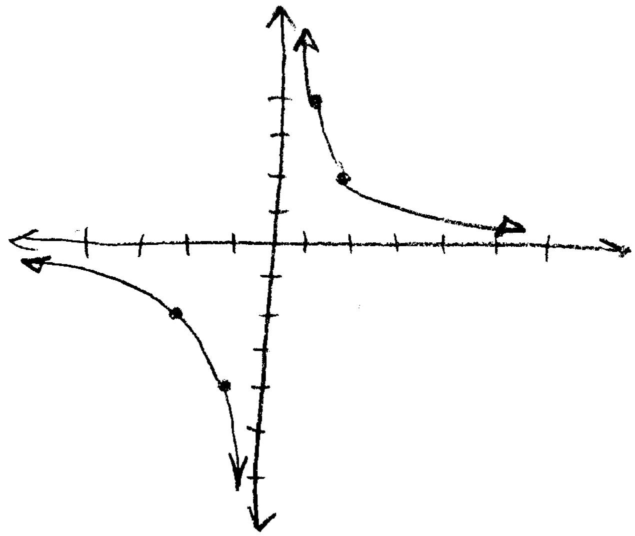


A special case of a hyperbola:

$$\boxed{xy = c}$$

Ex $xy = 4$

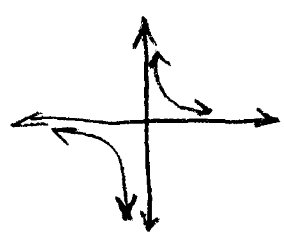
x	$y = \frac{4}{x}$
-2	$\frac{4}{-2} = -2$
-1	$\frac{4}{-1} = -4$
0	UNDEFINED
1	4
2	2
-10	$\frac{4}{-10} = -\frac{2}{5}$
$-\frac{1}{2}$	$\frac{4}{-\frac{1}{2}} = -8$
10	$\frac{4}{10} = \frac{2}{5}$



Asymptotes are the x and y axis.

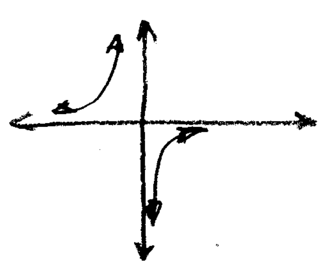
Ex $xy = 1$

$y = \frac{1}{x}$



Ex $xy = -1$

$y = -\frac{1}{x}$



Ch. 8-6 Conic Sections

Parabola $y = a(x-h)^2 + k$

Circle $(x-h)^2 + (y-k)^2 = r^2$

Ellipse $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

Hyperbola $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

See pg. 449

Looking at all the combinations of x^2, y^2, x^1, y^1 and numbers:

$$\boxed{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$$

Usually $B = 0$, the xy term "Rotates" the conic section, $xy = c$ is a special case of a hyperbola

Parabola $A = 0$ or $C = 0$

Circle $A = C$

Ellipse $A \neq C$ But both are + or both −

Hyperbola A and C have opposite signs.

Complete squares on x's or y's to put in standard form $= (h, k)$ form

Ex 2 Pg. 450 Parabola, circle, Ellipse, or Hyperbola?

(A) $y^2 - 2x^2 - 4x - 4y - 4 = 0$

$\boxed{A = 1 \quad C = -2 \quad \text{opposite signs, hyperbola}}$

(B) $4x^2 + 4y^2 + 20x - 12y + 30 = 0$

$\boxed{A = 4 \quad C = 4 \quad A = C \Rightarrow \text{circle.}}$

(C) $y^2 - 3x + 6y + 12 = 0$

$\boxed{A = 0 \ (\text{no } x^2 \text{ term}), \ C = 1 \quad \text{parabola}}$

More:

(d) $x^2 - 14x + 4 = 9y^2 - 36y$

Put in General Form

$x^2 - 9y^2 - 14x - 36y + 4 = 0$

$\boxed{A = 1 \quad C = -9 \quad \text{opp. signs} = \text{hyperbola}}$

(e) $5x^2 + 6x - 4y = x^2 - y^2 - 2x$

$-x^2 \quad +2x \qquad\qquad -x^2 \qquad\qquad +2x$

$4x^2 + y^2 + 8x - 4y = 0$

$\boxed{A = 4 \quad C = 1 \quad A \neq C \quad \text{Ellipse}}$

$\boxed{\text{BOTH } \oplus}$

There are special cases where what looks like a conic section "degenerates" into a point or a line.

Ex $x^2 + y^2 - 4x + 2y + 5 = 0$

$x^2 - 4x + 2^2 + y^2 + 2y + 1^2 = -5 + 4 + 1$

$(x-2)^2 + (y+1)^2 = 0$

circle with radius of zero = point at $(2, -1)$.

Homework: Pg. 450 # 4–9.