BE-Alg. 2 & PreCalc | Monday 9-21-09

"Real" ACT 2ND Ed. PT3

1. On the real number line, what is the midpoint of -5 and 17?

2. If $3\frac{3}{5} = x + 2\frac{2}{3}$, then $x = ?$

3. A system of linear equations is shown below.

$$3y = -2x + 8$$
$$3y = 2x + 8$$

Which of the following describes the graph of this system of linear equations in the standard (x, y) coordinate plane?

A. Two distinct intersecting lines.
B. Two parallel lines with positive slope.
C. Two parallel lines with negative slope.
D. A single line with positive slope.
E. A single line with negative slope.

- Time permitting ⇒ review HW 5 & Quiz 4
- Best practice problems ⇒ Real ACT
- Best topic-by-topic study guide ⇒ Barron's

BE Answers

① Mid $\Rightarrow \dfrac{-5+17}{2} = \boxed{6}$

② $3\frac{3}{5} = x + 2\frac{2}{3}$

$15 \cdot \dfrac{18}{5} = \left(x + \dfrac{8}{3}\right) \cdot 15$

$54 = 15x + 40$

$14 = 15x$

$\boxed{\dfrac{14}{15} = x}$

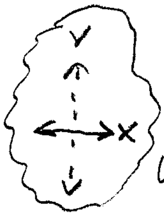
③ $3y = -2x + 8 \qquad y = -\frac{2}{3}x + \frac{8}{3}$

$3y = 2x + 8 \qquad y = \frac{2}{3}x + \frac{8}{3}$

$\boxed{\text{two distinct intersecting lines}}$

Functions of Two Variables

$y = f(x) = 2x + 1$ is a function "of"



one variable, x. The domain is all allowable values of x. The range is all allowable values of $2x+1$. Each x corresponds with <u>one</u> y.

$$x \rightarrow \boxed{\text{"f" function}} \rightarrow y$$

$z = f(x,y) = 2x + 3y$ is a function "of" <u>two</u> variables, x and y. The domain is all allowable values of x and y. Or in graphing terms, any allowable (x,y) pair. The range is all allowable values of $2x+3y$. Each (x,y) corresponds with <u>one</u> z.

$$(x,y) \rightarrow \boxed{\text{"f" function}} \rightarrow z$$

Ex) Find $f(4,5)$

$$(4,5) \rightarrow \boxed{2(4)+3(5)} \rightarrow 23$$

$$f(4,5) = 23.$$

<u>OR</u>, the ordered triple is $(4, 5, 23)$
x, y, z

You have often used functions of two variables although you may not have thought of them that way.

Ex) The area of a rectangle formula is an area function of two variables base and height.

$$A = A(b,h) = bh$$

Ex) Find $A(4,6)$

$$A(b,h) = bh$$

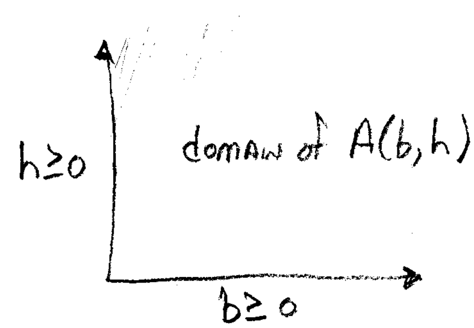
$$\boxed{A(4,6) = (4)(6) = 24 \text{ units}^2}$$

Are there any restrictions on the domain of this function?

$b \geq 0$ Since length $\neq$ negative

$h \geq 0$

$\therefore$ (b,h) must be in Quadrant I



Summary – how to find the value of any function of two variables?

Same way as you do a function of 1 variable.

- Rewrite original function
- Directly below it substitute the domain

Ex) $f(x,y) = 5x - y \qquad g(x,y) = x + y$

(A) Find $f(1,-6)$

$f(x,y) = 5x - y$

$f(1,-6) = 5(\ ) - (\ )$ You would do

$= 5(1) - (-6)$ this on 1 line

$= 5 + 6$

$\boxed{f(1,-6) = 11}$

(B) Find $g(-2,0)$

$g(x,y) = x + y$

$g(-2,0) = (-2) + (0)$

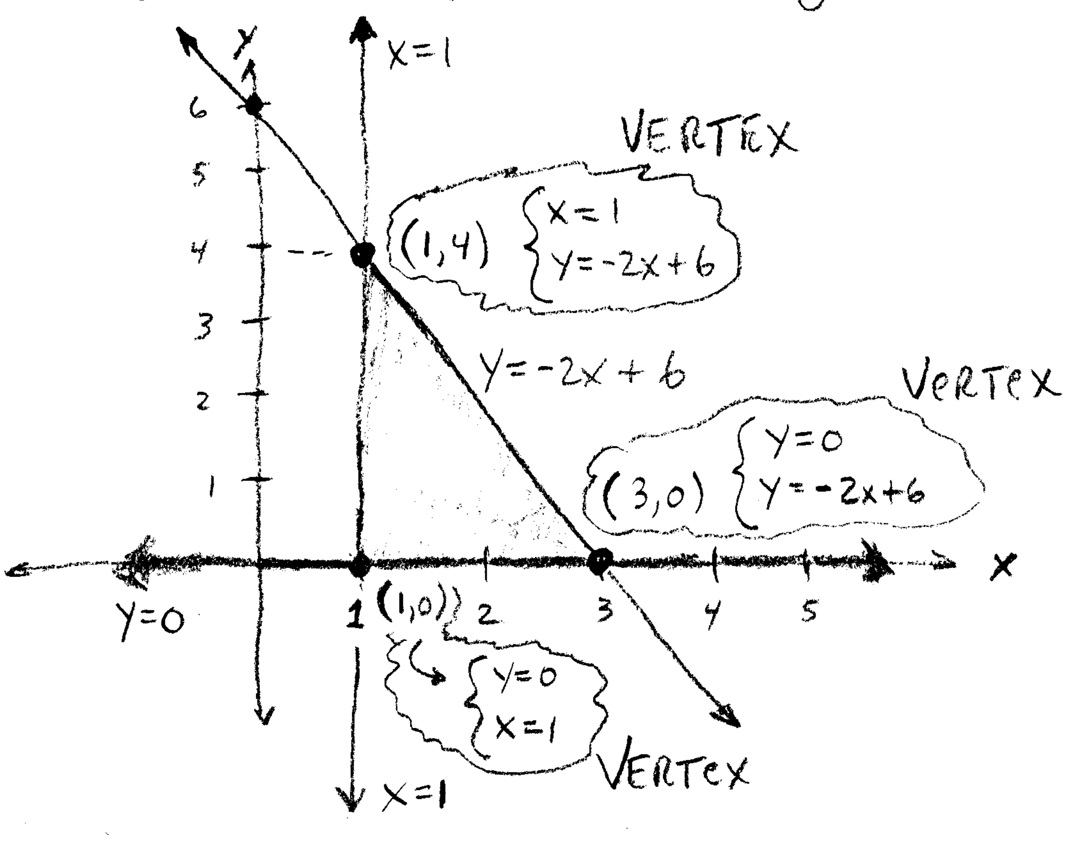
$\boxed{g(-2,0) = -2}$

VERTEX — THE POINT (x, y) formed by the intersection of 2 LINES or LINE SEGMENTS.

VERTICES ⇒ PLURAL

(EX) (Ex 1 Pg 129) The following system of INEQUALITIES forms 3 vertices WHEN THE boundary LINES ARE GRAPHED:

$$\begin{cases} x \geq 1 & \Rightarrow \text{vertical line at } x \geq 1 \\ y \geq 0 & \Rightarrow \text{horizontal line at } y \geq 0 \text{ i.e. the x-axis} \\ 2x + y \leq 6 & \Rightarrow \text{diagonal line at } y \leq -2x + 6 \end{cases}$$



Now, suppose you have a function of two variables whose domain (feasible region) is a set of (x, y) pairs in a shaded region defined by the graph of a system of linear inequalities (also called the "constraints").

This domain, if enclosed on all sides is called "bounded", if one side goes to ∞ it is called "unbounded"

A basic property of functions of 2 variables that have a domain that is bounded or unbounded by a system of linear inequalities is that <u>if</u> the function has a maximum or minimum value, it <u>will</u> be at one of the <u>vertices</u>!

Finding this max or min = Linear Programming = Ch. 3-4.

Steps to solve a linear programming problem $\Rightarrow$

- Graph the system of linear inequalities
- Use EBS or EBA to find the coordinates of each vertex.
- Find $f(x,y)$ at each vertex and pick max or min as needed.

Continue Example 1 $\Rightarrow$ given $f(x,y) = 3x+y$

Find the max and min values of the function.

$\Rightarrow$ Find $f(x,y)$ at each vertex

(x,y)	$f(x,y) = 3x+y$	
$(1,4)$	$3(1)+(4) = 7$	
$(3,0)$	$3(3)+(0) = 9$	$\Rightarrow$ MAX
$(1,0)$	$3(1)+(0) = 3$	$\Rightarrow$ MIN

$f(x,y) = 9$ = Maximum

$f(x,y) = 3$ = Minimum

Homework:
- Read 3-4
- Pg. 132 # 3, 5 (Tip: 4 vertices)