BE-Alg. 2 Monday 11-30-09

(1.) Define the inverse of a function.

(2.) Is the inverse of a function always a function? If not, how can you tell if it is?

(3.) $y = f(x) = x + 3$ find $f^{-1}(x)$
Prove it is $f^{-1}(x)$

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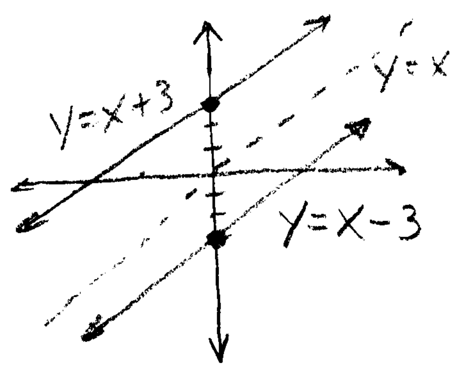
(3) ans $\Rightarrow$ $y = x + 3 = f(x)$

$x = y + 3$

$\boxed{x - 3 = y = f^{-1}(x)}$



$f(x) = x + 3$

$f(x-3) = (x-3) + 3$
$= x$ ✓

$(f \circ f^{-1})(x) = x$ ✓

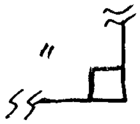
$f^{-1}(x) = x - 3$

$f^{-1}(x+3) = (x+3) - 3$
$= x$ ✓

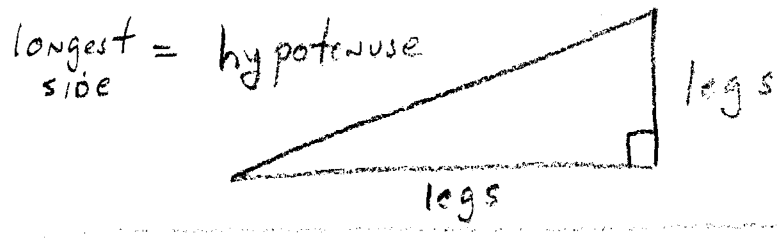
$(f^{-1} \circ f)(x) = x$ ✓

Ch. 13-1 Right Triangle Trigonometry

Right Triangle Vocabulary and Conventions & Properties

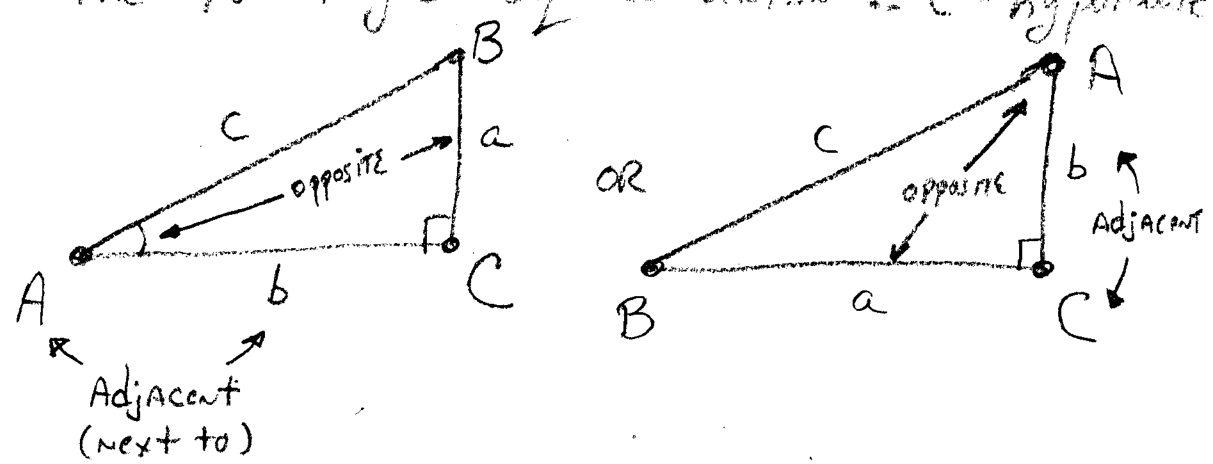


" " 90° symbol (Ex) 

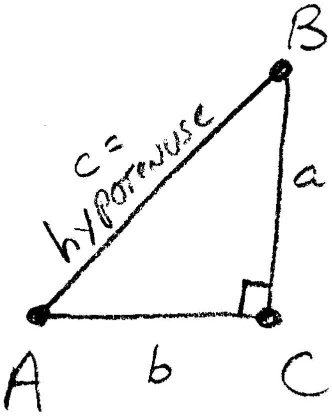


Every right (90°) triangle has ONE 90° Angle and two Acute (<90°) Angles. The two Acute Angles Always add to 90° and together with the 90° Angle = 180°

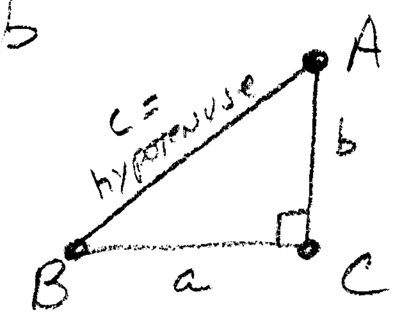
A, B, C = Vertices or the Angles (degrees)
a, b, c = sides (length)
C = the 90° Angle by convention ∴ c = hypotenuse



- Adjacent and opposite are never used to describe the hypotenuse. The hypotenuse is called the... hypotenuse.

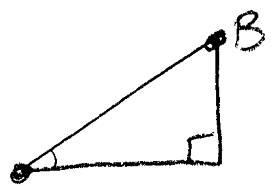


- ∠A is always opposite side a (and adjacent to side b) (Angle A)
- ∠B is always opposite side b (and adjacent to side a)

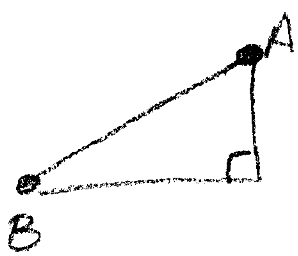


In any right triangle, once you know <u>ONE</u> acute angle - you know all 3 angles.

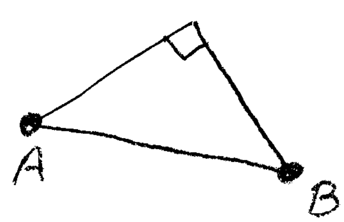
(EX) Name the missing ∠ value



A = 40°
B = ?
C = ?



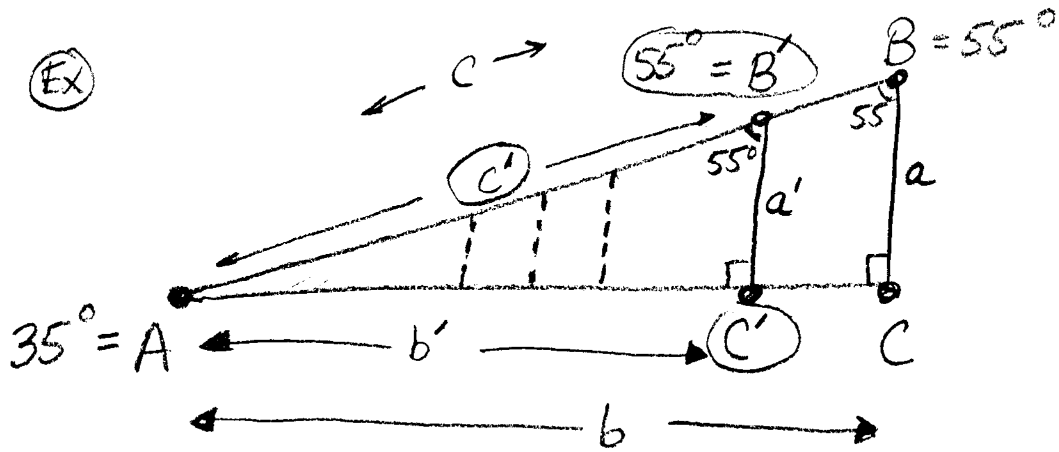
A = 10°
B = ?
C = ?



A = 45°
B = ?
C = ?

Because knowing the value of 1 acute angle "fixes" the value of the other two angles, for any given acute angle, say 35°, the ratio of side lengths of all 35°, 55°, 90° triangles are the same:

Ex



- infinite number of 35°-55°-90° triangles but the ratio of sides is always the same number

For $A = 35°$, $\frac{a}{c} = \frac{a'}{c'} = \frac{\text{any } a}{\text{any } c} \cong .5736$

$$\frac{b}{c} = \frac{b'}{c'} = \frac{\text{any } b}{\text{any } c} \cong .8192$$

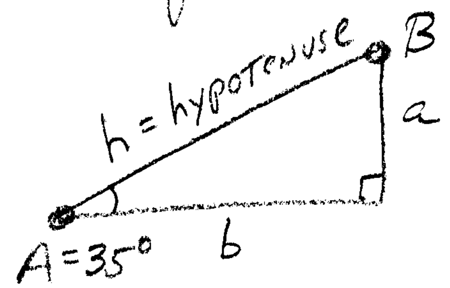
$$\frac{a}{b} = \frac{a'}{b'} = \frac{\text{any } a}{\text{any } b} \cong .7002$$

$\frac{\text{rise}}{\text{run}}$ of A = slope, slope of 45° = 1

slope of 35° = < 1 but close

Since, for any particular acute angle in a right triange the ratio of the lengths of the sides are always the same, we name and define them.

For this example:



$$\text{SINE} = \sin A = \sin 35^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h} \cong .5736 \quad \text{(SOH)}$$

$$\text{COSINE} = \cos A = \cos 35^\circ = \frac{\text{Adjacent}}{\text{hypotenuse}} = \frac{a}{h} \cong .8192 \quad \text{(CAH)}$$

$$\underset{\text{(slope)}}{\text{tangent}} = \tan A = \tan 35^\circ = \frac{\text{opposite}}{\text{Adjacent}} = \frac{o}{a} \cong .7002 \quad \text{(TOA)}$$

For any right triangle, where θ (theta = greek letter) is an acute $\angle$

$$\sin\theta = \frac{o}{h} \qquad \cos\theta = \frac{a}{h} \qquad \tan\theta = \frac{o}{a}$$

$$\boxed{\text{SOH} \qquad \text{CAH} \qquad \text{TOA}}$$

Find: sin, cos, tan 35° from: table (handout)
calcutator
(bring ACT calculator)

SOH CAH TOA → sin=o/h | cos=a/h | tan=o/a = rise/run
tan(0)=0, tan(90) = undefined, tan(45) = 1

Table : Trigonometric Functions

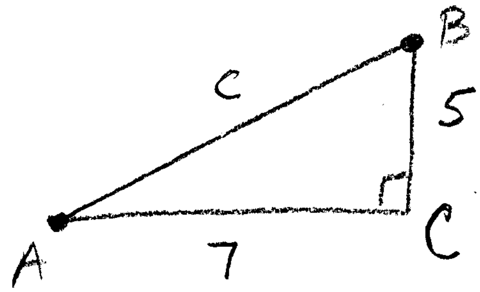
Use for angles between 0 and 45 degrees.

Degrees	Radians	Sin	Cos	Tan	Cot		
0	0.0000	0.0000	1.0000	0.0000		1.5708	90
1	0.0175	0.0175	0.9998	0.0175	57.290	1.5533	89
2	0.0349	0.0349	0.9994	0.0349	28.636	1.5359	88
3	0.0524	0.0523	0.9986	0.0524	19.081	1.5184	87
4	0.0698	0.0698	0.9976	0.0699	14.301	1.5010	86
5	0.0873	0.0872	0.9962	0.0875	11.430	1.4835	85
6	0.1047	0.1045	0.9945	0.1051	9.5144	1.4661	84
7	0.1222	0.1219	0.9925	0.1228	8.1443	1.4486	83
8	0.1396	0.1392	0.9903	0.1405	7.1154	1.4312	82
9	0.1571	0.1564	0.9877	0.1584	6.3138	1.4137	81
10	0.1745	0.1736	0.9848	0.1763	5.6713	1.3963	80
11	0.1920	0.1908	0.9816	0.1944	5.1446	1.3788	79
12	0.2094	0.2079	0.9781	0.2126	4.7046	1.3614	78
13	0.2269	0.2250	0.9744	0.2309	4.3315	1.3439	77
14	0.2443	0.2419	0.9703	0.2493	4.0108	1.3265	76
15	0.2618	0.2588	0.9659	0.2679	3.7321	1.3090	75
16	0.2793	0.2756	0.9613	0.2867	3.4874	1.2915	74
17	0.2967	0.2924	0.9563	0.3057	3.2709	1.2741	73
18	0.3142	0.3090	0.9511	0.3249	3.0777	1.2566	72
19	0.3316	0.3256	0.9455	0.3443	2.9042	1.2392	71
20	0.3491	0.3420	0.9397	0.3640	2.7475	1.2217	70
21	0.3665	0.3584	0.9336	0.3839	2.6051	1.2043	69
22	0.3840	0.3746	0.9272	0.4040	2.4751	1.1868	68
23	0.4014	0.3907	0.9205	0.4245	2.3559	1.1694	67
24	0.4189	0.4067	0.9135	0.4452	2.2460	1.1519	66
25	0.4363	0.4226	0.9063	0.4663	2.1445	1.1345	65
26	0.4538	0.4384	0.8988	0.4877	2.0503	1.1170	64
27	0.4712	0.4540	0.8910	0.5095	1.9626	1.0996	63
28	0.4887	0.4695	0.8829	0.5317	1.8807	1.0821	62
29	0.5061	0.4848	0.8746	0.5543	1.8040	1.0647	61
30	0.5236	0.5000	0.8660	0.5774	1.7321	1.0472	60
31	0.5411	0.5150	0.8572	0.6009	1.6643	1.0297	59
32	0.5585	0.5299	0.8480	0.6249	1.6003	1.0123	58
33	0.5760	0.5446	0.8387	0.6494	1.5399	0.9948	57
34	0.5934	0.5592	0.8290	0.6745	1.4826	0.9774	56
35	0.6109	0.5736	0.8192	0.7002	1.4281	0.9599	55
36	0.6283	0.5878	0.8090	0.7265	1.3764	0.9425	54
37	0.6458	0.6018	0.7986	0.7536	1.3270	0.9250	53
38	0.6632	0.6157	0.7880	0.7813	1.2799	0.9076	52
39	0.6807	0.6293	0.7771	0.8098	1.2349	0.8901	51
40	0.6981	0.6428	0.7660	0.8391	1.1918	0.8727	50
41	0.7156	0.6561	0.7547	0.8693	1.1504	0.8552	49
42	0.7330	0.6691	0.7431	0.9004	1.1106	0.8378	48
43	0.7505	0.6820	0.7314	0.9325	1.0724	0.8203	47
44	0.7679	0.6947	0.7193	0.9657	1.0355	0.8029	46
45	0.7854	0.7071	0.7071	1.0000	1.0000	0.7854	45
		Cos	Sin	Cot	Tan	Radians	Degrees

Use for angles between 45 and 90 degrees.

SOHCAHTOA

(EX) FIND SIN, COS, TAN for $\angle A$ AND $\angle B$
(find EXACT value = simplified fraction, then compare with value from table & calculator)



$$c^2 = a^2 + b^2 \quad \therefore \quad c^2 = 49 + 25$$

$$c = \sqrt{74} \sim 8.6023$$

$$\sin A = \frac{o}{h} = \underset{\text{EXACT}}{\boxed{\frac{5}{\sqrt{74}}}} = \frac{5}{\sqrt{74}} \cdot \frac{\sqrt{74}}{\sqrt{74}} = \underset{\text{EXACT}}{\boxed{\frac{5\sqrt{74}}{74}}}$$

"RATIONALIZED"
NO $\sqrt{\ }$ in bottom

$$\cong \frac{5}{8.6023} \cong \boxed{.5812} \text{ Approximate}$$

$$\cos A = \frac{a}{h} = \frac{7}{\sqrt{74}} = \underset{\text{EXACT}}{\boxed{\frac{7\sqrt{74}}{74}}} \cong \frac{7}{8.6023} = \underset{\text{Approximate}}{\boxed{.8137}}$$

$$\tan A = \frac{o}{a} = \underset{\text{EXACT}}{\boxed{\frac{5}{7}}} \cong \underset{\text{APPROXIMATE}}{\boxed{.7142}}$$

$$\sin B = \frac{7}{\sqrt{74}} \qquad \cos B = \frac{5}{\sqrt{74}}$$

See above For EXACT AND Approximate

* NOW CAREFUL $\tan B = \text{SLOPE } B = \frac{o}{a} = \underset{\text{EXACT}}{\boxed{\frac{7}{5}}} = \underset{\text{EXACT}}{\boxed{1.4}}$ WHY is ONE tan > 1 AND ONE is < 1 ?

If $\sin A \cong .5812$ CAN you FIND $\angle A$?

Using table $\Rightarrow$ find A SIN VALUE OF .5812 (closest to it) And read Angle $\cong$ 36°

The process of going backward from the value of the sin to the angle is called finding the inverse of the sin $\Rightarrow \sin^{-1} A$

OR sometimes it is called the ARC SIN A $\therefore \sin^{-1}(.5812) = \arcsin(.5812) = 36°$

$\therefore \angle B = 90 - 36 = 54°$

The "reciprocal" trig. functions

$\sin\theta = \frac{O}{H}$ $\quad \boxed{\csc\theta = \frac{H}{O}}$ $\quad$ Cosecant

OR $\frac{1}{\sin\theta} = \frac{1}{\frac{O}{H}} = \frac{H}{O}$

$\cos\theta = \frac{a}{H}$ $\quad \boxed{\sec\theta = \frac{H}{a}} =$ Secant

OR $\frac{1}{\cos\theta} = \frac{1}{\frac{a}{H}} = \frac{H}{a}$

$\tan\theta = \frac{O}{A}$ $\quad \boxed{\cot\theta = \frac{A}{O}}$ $\quad$ Cotangent

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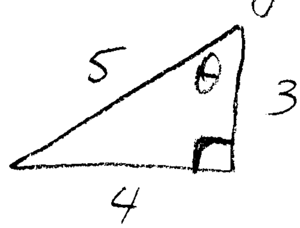
Ex 1 Pg 702 Find all 6 trig. functions for θ



$\sin\theta = \frac{4}{5} \quad \therefore \quad \csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5} \quad \therefore \quad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3} \quad \therefore \quad \cot\theta = \frac{3}{4}$

Homework: Pg. 706 # 2, 4–6, 15