PRECALC-BE Tuesday 1-5-10

FIND THE PATTERN:

① 4, 12, 20, ...

② −61, −56, −51, ...

③ 17, 11, 5, ...

④ 22, 33, 37, ...

SEQUENCE A set of numbers arranged in some specified way.

May be finite (ex) {1, 2, 3}

or infinite (ex) {1, 2, 3, ...}

Arithmetic sequence A sequence in which there is a  common difference between each term.

① $d = 8$ ② $d = 5$ ③ $d = -6$ ④ NOT AN ARITH. SEQ.

Lets look at: $4, 12, 20, \cdots$

Name the terms $a_1, a_2, a_3, \cdots$

The common difference is $+8$

$\Rightarrow a_1,\ a_1+8,\ a_2+8,\ \cdots\ a_{N-1}+8$

a_1 a_2 a_3 a_N

1st term, 2nd term, 3rd term, ... N^{th} term

OR $a_1,\ a_1+1d,\ a_1+2d,\ \cdots\ a_1+(N-1)d$

$\therefore \boxed{a_N = a_1 + (N-1)d}$

N^{th} term = first term + (N^{th} term minus one) × common difference

*You do NOT ADD d to the first term to get the first term!

Formula for the N^{th} Term of Any Arithmetic Sequence

(Ex) Given $4, 12, 20, \ldots$ find the (A) 4th term, (B) 101st term

(A) $a_4 = 4 + (4-1)8 = 4 + (3)8 = \boxed{28}$ ✓

(B) $a_{101} = 4 + (101-1)8 = \boxed{804}$

If d is positive, sequence ↑

If d is negative, sequence ↓

Ch. 12-1 Arithmetic Sequences and Series

(EX 1) (Pg 759) Find the next 4 terms of $-5, -2, 1, \ldots$

$d = 3$ ✓ $d = 3$ ✓ ✓

$a_4 = -5 + (4-1)3 = 4 = a_4$

$a_5 = -5 + (5-1)3 = 7 = a_5$

$a_6 = -5 + (6-1)3 = 10 = a_6$

$a_7 = -5 + (7-1)3 = 13 = a_7$

$$\boxed{a_N = a_1 + (N-1)d}$$

↑ ↑ ↑ ↑

4 variables

∴ if you know ANY 3 of 4 you can solve for the ONE you are missing.

(Ex 3) (Pg 760) Given Arithmetic Sequence

$a_{19} = 42 \quad d = -\frac{2}{3}$

Find a_1

$a_N = a_1 + (N-1)d$

$\therefore \quad 42 = a_1 + (19-1)-\frac{2}{3}$

{Since a_{19} means $N = 19$}

$42 = a_1 + 18\left(-\frac{2}{3}\right)$

$42 = a_1 - 12$

$54 = a_1$

In an arithmetic sequence like

(Ex) $-5, \underset{\uparrow}{-2}, 1, 4, 7, \underset{\uparrow}{10}, 13, \ldots$

↑ ↑ ↑

Arithmetic means of $-2, 10$

The terms between any two non-consecutive terms of an arithmetic sequence are called **arithmetic means**.

(Ex) $4, 8, 12, 16, 20, \ldots$

The arithmetic mean of $8, 16$ is 12.

EX 4 PG 761 WRITE AN ARITHMETIC SEQUENCE THAT HAS 5 ARITHMETIC MEANS between 4.9 AND 2.5.

$4.9, \underline{?}, \underline{?}, \underline{?}, \underline{?}, \underline{?}, 2.5$

a_1 $\qquad\qquad$ a_7

$$a_N = a_1 + (N-1)d$$

$$2.5 = 4.9 + (7-1)d$$

$$2.5 = 4.9 + 6d$$

$$-4.9 \quad -4.9$$

$$-2.4 = 6d$$

$$\frac{-2.4}{6} = d \qquad \therefore \boxed{d = -.4}$$

$$\therefore \boxed{4.9, 4.5, 4.1, 3.7, 3.3, 2.9, 2.5}$$

ARITHMETIC SERIES: the sum of the terms of A finite ARITHMETIC series.

For INFINITE ARITHMETIC series you will need to specify A finite number of terms to add, this is called a partial sum.

(EX)

ARITHMETIC Sequence $3, 5, 7, 9$

ARITHMETIC Series $3+5+7+9$

Let S_N be the partial sum of AN ARITHMETIC series when N is the number of terms to add

$S_3 = 3+5+7 = 15$ OF course THERE IS A FORMULA

$$S_N = \frac{N}{2}(a_1 + a_N)$$ FORMULA FOR THE SUM OF A FINITE ARITHMETIC SERIES

$S_3 = \frac{3}{2}(3+7) = \frac{30}{2} = 15$ ✓

(See Pg 761 to see WHERE THIS comes FROM)

EX 5 Pg 761 Find the sum of the first 60 terms of $9+14+19+\ldots 304$.

$$S_N = \frac{N}{2}(a_1 + a_N)$$

$$S_N = \frac{60}{2}(9 + 304)$$

$$S_N = 30(313)$$

$$S_N = 9390$$

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* Homework: Pg 763
  # 6 – 16

+ Memorize formulas & definitions covered today!

Tip: Flash cards or summary pages.