PRECALC - BE | TUESDAY 1-19-10

(1.) New STUDENT AT college, choice of 3 MATH COURSES, 2 SCIENCE, AND 2 humanities. ONE course per subject, how many course schedules are possible?

FCP Fundamental Counting Principle

If 2 independent events CAN OCCUR (they don't affect each other) p ways and q ways, the total number of possible events is $p \cdot q$

Total schedules $= 3 \cdot 2 \cdot 2 = 12$ schedules
(MATH) (SCIENCE) (humanities)

A tree diagram, or at least a partial one, can help to understand the arrangement of choices:

Choice 1	Choice 2	Choice 3	Outcomes
$Math_1$	$Science_1$	hum_1	$M_1S_1H_1$
		hum_2	$M_1S_1H_2$
	$Science_2$	hum_1	$M_1S_2H_1$
		hum_2	$M_1S_2H_2$
$Math_2$	$Science_1$	hum_1	$M_2S_1H_1$
		hum_2	$M_2S_1H_2$
	$Science_2$	hum_1	$M_2S_2H_1$
		hum_2	$M_2S_2H_2$
$Math_3$	$Science_1$	hum_1	$M_3S_1H_1$
		hum_2	$M_3S_1H_2$
	$Science_2$	hum_1	$M_3S_2H_1$
		hum_2	$M_3S_2H_2$

"Random" Probability $(M_3S_2H_2) = \frac{1}{12}$

$P(M_3S_2H_2) = \frac{1}{12}$

independent events – EVENTS THAT DO NOT AFFECT EACH OTHER
dependent events – EVENTS THAT DO AFFECT EACH OTHER

Ch 13-1 Permutations and Combinations

Combinatorics

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permutation – AN arrangement of objects where order IS important

Combination – AN arrangement of objects where order IS NOT important.

Permutation ⇒ books on a shelf say, A, B, C, D

How many ways to arrange?

? ? ? ?

4 choices, 3 choices, 2 choices, 1 choice

$FCP \Rightarrow 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$4!$ read 4 factorial $= 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Careful: WHAT IS: 0!

For reasons THAT will be clearer later, it is important to <u>define</u> $0! = 1$.

Books on a shelf ⇒ order important
⇒ lots of arrangements.

(Ex) 20 STUDENTS IN Precalc class, assume "only" 20 chairs.
How many seating charts possible?

Guess? ____

$\underset{\uparrow}{20}, \underset{\uparrow}{19}, \underset{\uparrow}{18}, \underset{\uparrow}{17} \ \ldots$
choices

$20! = 2{,}432{,}902{,}008{,}000{,}000{,}000.0$
$\cong 2.4 \times 10^{18}$ seating charts possible!

Suppose I only had 4 chairs! How many ways could I make a seating chart then?

$\underset{\substack{\uparrow \\ 20 \\ \text{choices}}}{\underline{\ ?\ }}, \underset{19}{\underline{\ ?\ }}, \underset{18}{\underline{\ ?\ }}, \underset{17}{\underline{\ ?\ }}$

$\Rightarrow 20 \cdot 19 \cdot 18 \cdot 17 = 116,280.$

This is called a permutation of 20 things taken 4 at a time.

ORDER IS IMPORTANT

Fancy formula $\boxed{{}_N P_r = \frac{N!}{(N-r)!}}$

Ex) ${}_{20}P_4 = \frac{20!}{(20-4)!} = \frac{20!}{16!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16!}}{\cancel{16!}}$

$= 116,280$

LOOK Note: ${}_{20}P_{20} = \frac{20!}{(20-20)!} = \frac{20!}{0!} = \frac{20!}{1} = 20!$

Suppose I have 20 STUDENTS and 4 chairs and I WANT to know how many ways the STUDENTS can sit – but I don't care about order. That is A, B, C, D = D, C, B, A

Well – how many arrangements are affected by order is how many ways can I arrange 4 objects

$\therefore$ $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$ = number of extra ways if order <u>IS</u> important.

<u>Combination</u> – Arrangement where order is NOT important

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Reduce $\frac{20!}{16!}$ by $4!$ $\Rightarrow$ $\frac{116,280}{24} = 4845$

$$\boxed{{}_N P_r = \frac{N!}{(N-r)!}}$$

ORDER MATTERS, MOST NUMBER OF ARRANGEMENTS

The permutation of N things taken r at a time.

$$\boxed{{}_N C_r = \frac{N!}{(N-r)!\,r!}}$$

Order not important, reduce number of arrangements by $\frac{1}{r!}$

The combination of N things taken r at a time.

Homework: Ch 13-1 Permutations & Combinations

Pg 843 # 5, 7-14.