

BE-PRECALC Monday 2-8-10

$$\textcircled{1} (3x-y)^2 = (3x-y)(3x-y) = ?$$

$$\textcircled{2} (3x-y)^3 = ?$$

$$\textcircled{3} (3x-y)^4 = ?$$

When you expand a binomial like $(x+y)^n$, a pattern emerges:

$$\begin{aligned}
 (x+y)^0 &= 1x^0y^0 = 1 \\
 (x+y)^1 &= 1x^1y^0 + 1x^0y^1 = x+y \\
 (x+y)^2 &= 1x^2y^0 + 2x^1y^1 + 1x^0y^2 = x^2 + 2xy + y^2 \\
 (x+y)^3 &= 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3 \\
 (x+y)^4 &= 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4 \\
 (x+y)^5 &= 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5x^1y^4 + 1x^0y^5
 \end{aligned}$$

PASCAL'S
TRIANGLE

$$\begin{array}{ccccccc}
 & & & & 1 & & & \\
 & & & & 1 & & 1 & \\
 & & & 1 & & 2 & & 1 \\
 & & 1 & & 3 & & 3 & & 1 \\
 & 1 & & 4 & & 6 & & 4 & & 1 \\
 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
 1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
 \end{array}$$

EX 1 Pg 802 Ch 12-6 - THE BINOMIAL THEOREM

Let's use the pattern to expand

binomials $\Rightarrow (x+y)^6$
 $\downarrow$
 use 7th row $\Rightarrow$

$$1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1x^0y^6$$

Is there a formula?

YES

If N is a positive integer:

$$(x+y)^N = x^N + Nx^{N-1}y + \frac{N(N-1)}{2!}x^{N-2}y^2 + \dots$$

$$\frac{N(N-1)(N-2)}{3!}x^{N-3}y^3 + \dots + y^N$$

EX 3
Pg 803

Expand $(2x-y)^6$

$N=6$, there are 7 terms.

WATCH OUT $\Rightarrow 1, 6, \frac{6 \cdot 5}{2}, \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1}, ?, ?$
 $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 $1, 6, 15, 20, 15, 6, 1$

$(2x)^6 = 2^6 x^6$

$$2^6 x^6 - 6 \cdot 2^5 x^5 y + 15 \cdot 2^4 x^4 y^2 - 20 \cdot 2^3 x^3 y^3 + 15 \cdot 2^2 x^2 y^4 - 6 \cdot 2^1 x y^5 + y^6$$

$$64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$$

If you only want one particular term, say the 4th term of $(2x-y)^6$,

Use:
$$(X+Y)^N = \sum_{r=0}^N \frac{N!}{r!(N-r)!} X^{N-r} Y^r$$

Pg 803

↑
ANOTHER FORM
OF THE
BINOMIAL
THEOREM

↑
Sigma or "Sum" NOTATION

(EX)
$$\sum_{N=3}^5 N = 3+4+5 = 12$$

↑
STARTING "index"

Ex) 4th term of $(2x-y)^6$ Let $r=3$
 $N=6$, $r=\underline{3}$ since terms $\Rightarrow 0, 1, 2, 3, \dots$

$$(X+Y)^6 = \sum_{r=0}^6 \frac{6!}{r!(6-r)!} (2x)^{6-r} (-y)^r$$

$$\therefore r=3 \Rightarrow \frac{6!}{3!(3!)} (2x)^3 (-y)^3$$

$$= \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} \cdot 2^3 x^3 \cdot -1 y^3$$

$$= -20 \cdot 8 x^3 y^3$$

$$= \boxed{-160 x^3 y^3} \quad \checkmark$$

Ex 4
Pg 803 Find the fifth term of $(4a + 3b)^7$

$$N = 7 \quad r = 4 \quad \uparrow$$

$$(4a + 3b)^7 \Rightarrow \frac{7!}{4!(3)!} (4a)^{7-4} (3b)^4$$

FIFTH TERM

$$\frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} \cdot 4^3 a^3 \cdot 3^4 b^4$$

$$35 \cdot 64 a^3 \cdot 81 b^4$$

$$(35 \cdot 64 \cdot 81) a^3 b^4$$

$$\boxed{181,440 a^3 b^4} = 5^{\text{th}} \text{ term of } (4a + 3b)^7$$

Homework: Pg 804 # 7, 8, 9