BE - Precalc Wednesday 3-3-10

(1) Define an ellipse as "the set of points..."

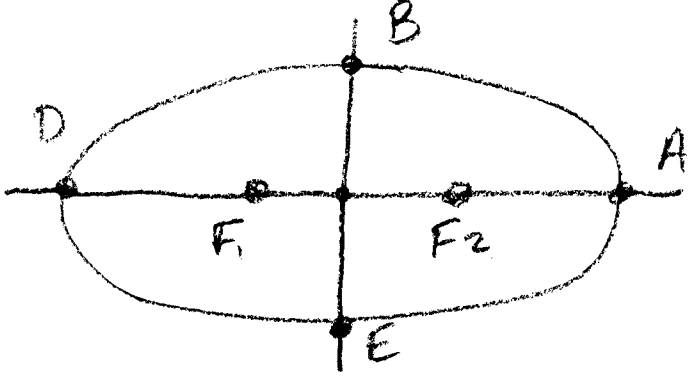
(2) Why did our string construction result in an ellipse?

Continue Ch 10-3, Ellipse

An ellipse has 2 axes of symmetry

major axis (longer) $\Rightarrow \overline{AD}$

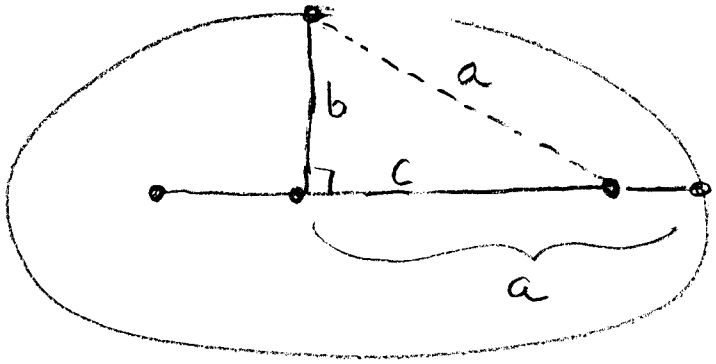
minor axis (shorter) $\Rightarrow \overline{BE}$



A, B, D, E are Vertices (end points of the axes)

$\frac{1}{2}$ of major axis = semi-major axis $= a$

$\frac{1}{2}$ of minor axis = semi-minor axis $= b$



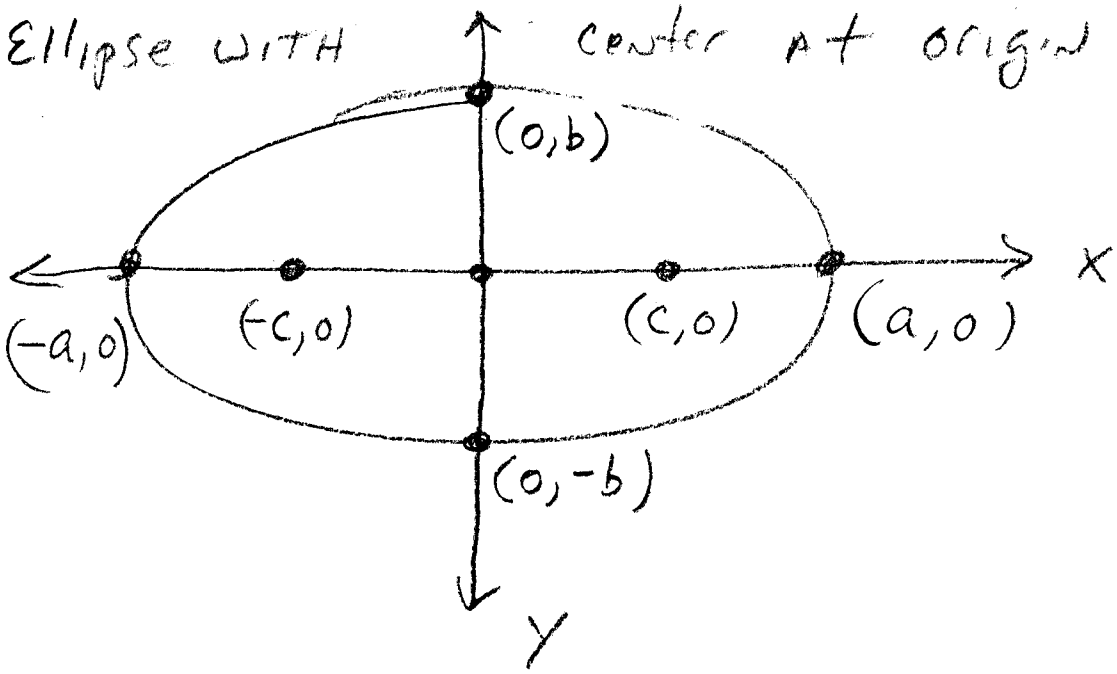
$c^2 + b^2 = a^2$

↑ midpoint to Focus

∴ Distance Between Foci $= 2c$

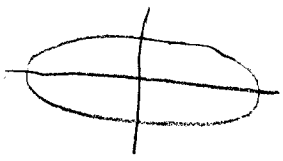
See Pg 632

Ellipse WITH center at origin



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

h, k form where h ⇒ horizontal SHIFT
k ⇒ vertical SHIFT
(of center)



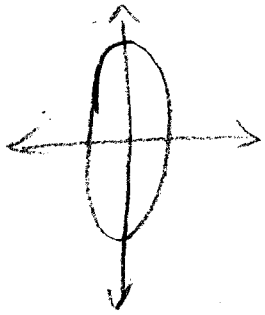
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

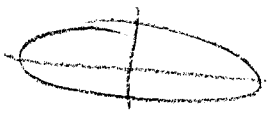
NOT

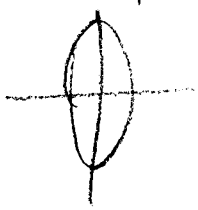
Center (h, k) MINOR AXIS $x = h$

$c^2 = a^2 - b^2$ MAJOR AXIS $y = k$

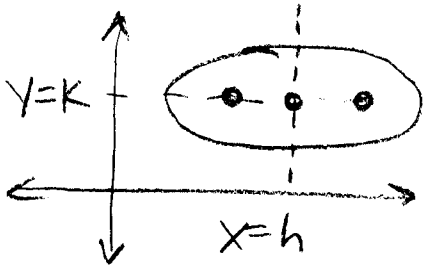
See TABLE Pg 634 $a^2 > b^2$ or c^2 *

What about  Ellipse?

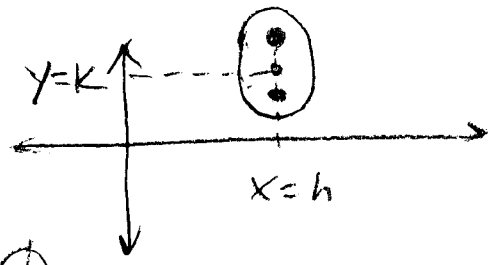
Compare: Is the biggest term $\Rightarrow a^2$ under the $(x-h)^2$? if so 

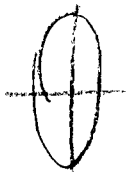
If under $(y-k)^2$? $\Rightarrow$ 

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



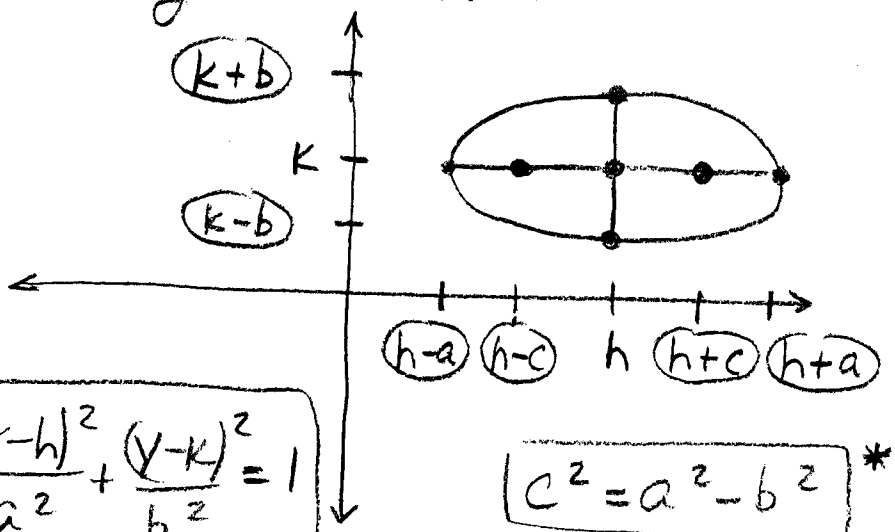
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$





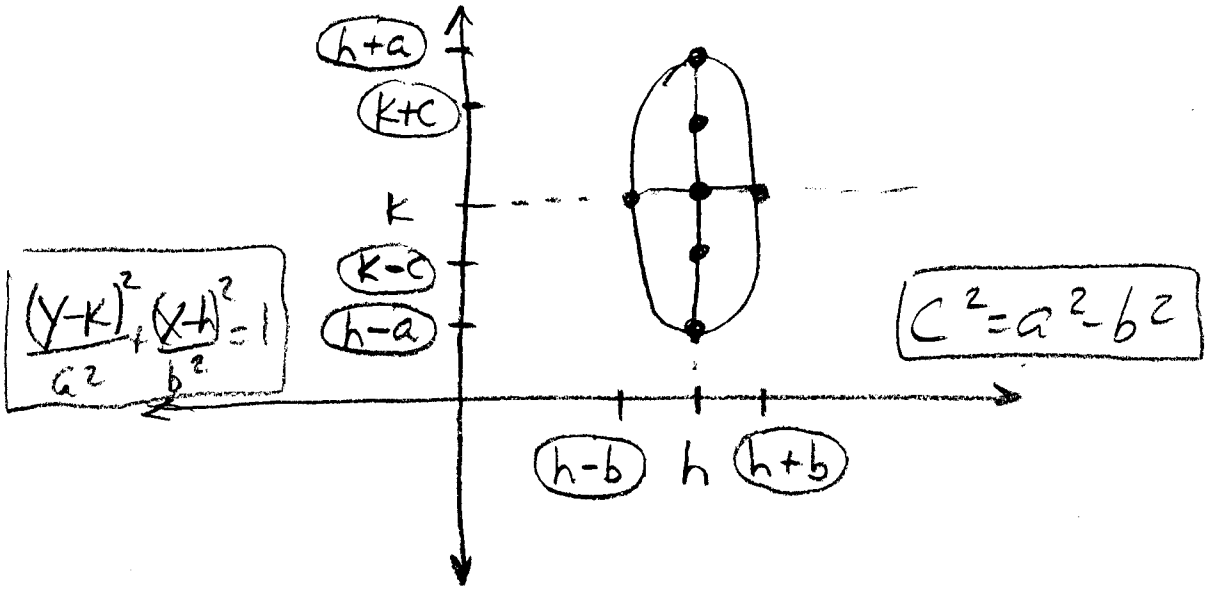
Center (h,k)	Center (h,k)
Foci $(h\pm c, k)$ ↑ major axis $y=k$, minor axis $x=h$	Foci $(h, k\pm c)$ ↑ major axis $x=h$, minor axis $y=k$
Major Vertices $(h\pm a, k)$	Major Vertices $(h, k\pm a)$
Minor Vertices $(h, k\pm b)$	Minor Vertices $(h\pm b, k)$

Look again at a, b, c:



$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$c^2=a^2-b^2 \quad *$$



$$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

$$c^2=a^2-b^2$$

(Ex 4 Pg 635) Find center, 2 foci, 4 vertices of

$4x^2 + 9y^2 - 40x + 36y + 100 = 0$

$4x^2 - 40x \quad + 9y^2 + 36y \quad = -100$

$4(x^2 - 10x + \boxed{5^2}) + 9(y^2 + 4y + \boxed{2^2}) = -100 + \boxed{4(25)} + \boxed{9(4)}$

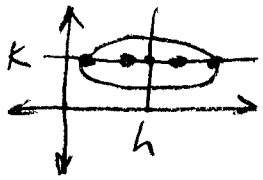
$\frac{4(x-5)^2}{36} + \frac{9(y+2)^2}{36} = \frac{36}{36}$ ← MUST BE 1 ∴ ÷ by 36

$\frac{(x-5)^2}{9} + \frac{(y+2)^2}{4} = 1$

↑ biggest ∴ $a^2 = 9 \Rightarrow a = 3$

$b^2 = 4 \Rightarrow b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5 \therefore c = \sqrt{5}$



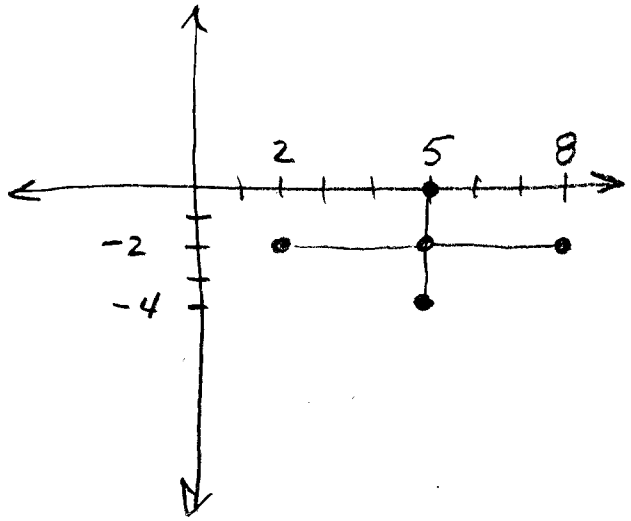
h, k

Center $(5, -2)$

Foci $(5 \pm \sqrt{5}, -2)$

V_{major} $(5 \pm 3, -2)$

V_{minor} $(5, -2 \pm 2)$



Homework: Pg 637 # 7, 8, 9.