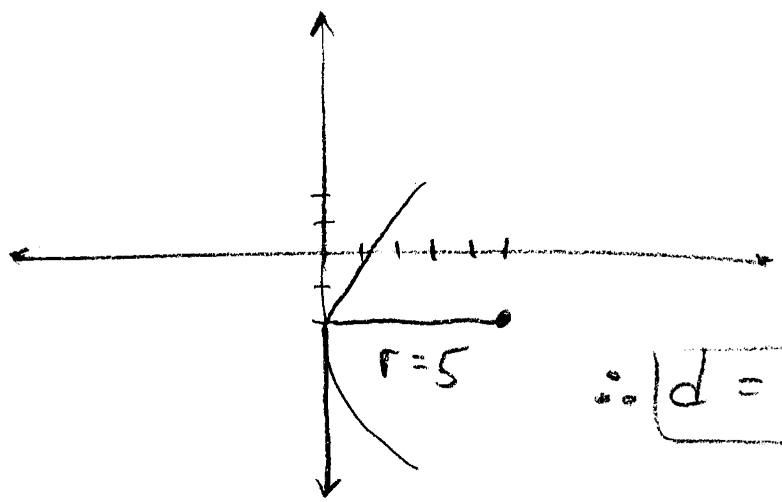
BE - PRECALCULUS MONDAY 11-9-09

ACT
Barron's

(1.) A circle WITH CENTER $(5,-2)$ is tangent to the y-AXIS. What is the diameter of the circle?

ANS!



$\therefore \boxed{d = 10 \text{ UNITS}}$

Mini-Review - 6 Basic Exponent Rules $\Rightarrow$ Shortcuts

MR $\Rightarrow$ Multiplying Powers With Same Base
$\Rightarrow$ Add Exponents

Ex $X^2(X^5) = XX(XXXXX) = \boxed{X^7}$

DR $\Rightarrow$ Dividing Powers With Same Base
Top > Bot $\Rightarrow$ Subtract Exponents (Top − Bottom)

Ex $\dfrac{X^5}{X^2} = \dfrac{\cancel{X}\cancel{X}XXX}{\cancel{X}\cancel{X}} = \boxed{X^3}$

ZER $\Rightarrow$ Ex $\dfrac{X^5}{X^5} = X^{5-5} = \boxed{X^0} = \dfrac{X^5}{X^5} = 1$

Top = Bot

NER $\Rightarrow$ Ex $\dfrac{X^2}{X^5} = \dfrac{\cancel{X}\cancel{X}}{\cancel{X}\cancel{X}XXX} = \dfrac{1}{X^3} = X^{2-5} = X^{-3}$

Top < Bot

PP $\Rightarrow$ Raising A Power To A Power
$\Rightarrow$ Multiply Exponents

Ex $(X^5)^2 = (XXXXX)(XXXXX) = \boxed{X^{10}}$

GP $\Rightarrow$ Raising A Group To A Power

$$\left(\frac{ab}{c}\right)^2 = \left(\frac{ab}{c}\right)\left(\frac{ab}{c}\right) = \frac{aabb}{cc} = \boxed{\frac{a^2b^2}{c^2}}$$

Can we use the 6 basic exponent rules if the exponents are fractions?

Yes!

Ex Since $\sqrt{x} \cdot \sqrt{x} = x$

And $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2}+\frac{1}{2}} = x^{1} = x$

Then $\sqrt{x} = x^{\frac{1}{2}}$ OR $x^{\frac{1}{2}} = \sqrt[2]{x^{1}} = \sqrt{x}$

$\therefore x^{\frac{1}{3}} = \sqrt[3]{x^{1}} = \sqrt[3]{x}$

etc.

Also: $x^{\frac{2}{3}} = \sqrt[3]{x^{2}} = \sqrt[3]{x \cdot x} = \sqrt[3]{x}\,\sqrt[3]{x}$

$= \left(\sqrt[3]{x}\right)^{2}$

Ex $x^{\frac{2}{5}} = \sqrt[5]{x^{2}}$ or $\left(\sqrt[5]{x}\right)^{2}$

Ex $x^{\frac{1}{2}} \cdot x^{\frac{2}{3}} = ? = x^{\frac{3}{6}} \cdot x^{\frac{4}{6}} = \boxed{x^{\frac{7}{6}}}$

Ex $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}} = x^{\frac{2}{3}-\frac{1}{3}} = \boxed{x^{\frac{1}{3}}}$

Ex $\left(x^{\frac{5}{6}}\right)^{2} = x^{\frac{10}{6}} = \boxed{x^{\frac{5}{3}}}$ or $\sqrt[3]{x^{5}}$ or $\left(\sqrt[3]{x}\right)^{5}$

Ex $\sqrt[7]{x^{3}} = ?$ As a fractional exponent? $\Rightarrow \boxed{x^{\frac{3}{7}}}$

WATCH OUT, you CAN simplify some RADICALS THAT DON'T look like they CAN be simplified by using FRACTIONAL EXPONENTS:

(Ex) $\sqrt[6]{8} = ?$

$\sqrt[6]{2^3} = 2^{\frac{3}{6}} = \boxed{2^{\frac{1}{2}} \text{ or } \sqrt{2}}$

Is this reasonable?

$\sqrt[6]{8} \overset{?}{=} \sqrt{2} \Rightarrow (\sqrt{2}\cdot\sqrt{2})(\sqrt{2}\cdot\sqrt{2})(\sqrt{2}\cdot\sqrt{2})$

$2 \cdot 2 \cdot 2 = 8$ ✓

The 7th Exponent Rule $\Rightarrow$ FR

$\Rightarrow$ Fractional Exponent Rule

$$\boxed{\text{FR} \quad a^{\frac{N}{m}} = \sqrt[m]{a^N} = \left(\sqrt[m]{a}\right)^N}$$

(Ex) $8^{\frac{2}{3}} = \sqrt[3]{8^2}$ OR $\left(\sqrt[3]{8}\right)^2$

$= \sqrt[3]{64}$ | $= (2)^2$ Since $2\cdot2\cdot2=8$

$= \boxed{4}$ | $= \boxed{4}$

Since $4\cdot4\cdot4=64$

* Simplifying RADICALS: MUST Extract Perfect SQUARE FACTORS

$\sqrt{1} = 1$ Perfect Square (PS)

$\sqrt{2} = \sqrt{2}$

$\sqrt{3} = \sqrt{3}$

$\sqrt{4} = 2$ PS

$\sqrt{5} = \sqrt{5}$

$\sqrt{6} = \sqrt{6}$

$\sqrt{7} = \sqrt{7}$

* $\sqrt{8} = \sqrt{8} = \sqrt{4 \cdot 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

$\sqrt{9} = 3$

$\sqrt{10} = \sqrt{10}$

$\sqrt{11} = \sqrt{11}$

* $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

$\sqrt{13} = \sqrt{13}$

$\sqrt{14} = \sqrt{14}$

$\sqrt{15} = \sqrt{15}$

$\sqrt{16} = 4$

$\sqrt{17} = \sqrt{17}$

* $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

$\sqrt{19} = \sqrt{19}$

* $\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

$\sqrt{21} = \sqrt{21}$

$\sqrt{22} = \sqrt{22}$

$\sqrt{23} = \sqrt{23}$

* $\sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

$\sqrt{25} = 5$

$\sqrt{26} = \sqrt{26}$

* $\sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

$\sqrt{28} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

$\sqrt{29}$

$\sqrt{30}$

$\sqrt{31}$

* $\sqrt{32} = \sqrt{16}\sqrt{2} = 4\sqrt{2}$

or $\sqrt{4}\sqrt{8} = 2\sqrt{8}$

$= 2\sqrt{4}\sqrt{2}$

$4\sqrt{2}$

$\sqrt{X} = \sqrt{X}$

$\sqrt{X^2} = X \Rightarrow$ Really $\sqrt[2]{X^2} = |X|$

INDEX

ALWAYS $\oplus$ ALWAYS $\oplus$

" $\underbrace{EI}_{\text{EVEN INDEX}}$ $\underbrace{EI}_{\text{EVEN EXPONENT INSIDE}}$ $\underbrace{OO}_{\text{ODD EXPONENT OUTSIDE}}$ " $\Rightarrow$ USE ABSOLUTE VALUE

* $\sqrt{X^3} = \sqrt{X^2}\sqrt{X} = X\sqrt{X}$

$\sqrt{X^4} = \sqrt{X^2}\sqrt{X^2} = XX = X^2$

* $\sqrt{X^5} = \sqrt{X^4}\sqrt{X} = X^2\sqrt{X}$

$\sqrt{X^6} = \sqrt{X^3}\sqrt{X^3} = |X^3|$ EIEIOO

* $\sqrt{X^7} = \sqrt{X^6}\sqrt{X} = X^3\sqrt{X}$

$\sqrt{X^8} = X^4$

$\sqrt{X^9} = \sqrt{X^8}\sqrt{X} = X^4\sqrt{X}$

$\sqrt{X^{10}} = |X^5|$ EIEIOO

Ch. 11-1 Real Exponents

Ex 4 pg 697

(A) $125^{\frac{1}{3}} = \sqrt[3]{125} = \boxed{5}$

or $125^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = 5^{\frac{3}{3}} = \boxed{5^1}$ ✓

Recognize this is 5^3

(B) $\sqrt{14} \cdot \sqrt{7} = \sqrt{98} = \sqrt{49}\sqrt{2} = \boxed{7\sqrt{2}}$

$98 \to 49 \; 2$

or $\sqrt{7} \cdot \sqrt{2} \cdot \sqrt{7} = \boxed{7\sqrt{2}}$

or $(2 \cdot 7)^{\frac{1}{2}} \cdot 7^{\frac{1}{2}} = 2^{\frac{1}{2}} 7^{\frac{1}{2}} 7^{\frac{1}{2}}$

$2^{\frac{1}{2}} \cdot 7 = \boxed{7\sqrt{2}}$ ✓

Ex 5 $(81c^4)^{\frac{1}{4}} = \sqrt[4]{81c^4} = \boxed{3|c|}$

E18100

or $81^{\frac{1}{4}} c^{\frac{4}{4}}$

$= (3^4)^{\frac{1}{4}} c^1$

$3|k|$

Ex 6 Pg 695 (A) $625^{\frac{3}{4}}$

$$\sqrt[4]{625^3} = \left(\sqrt[4]{625}\right)^3$$ MUCH EASIER

$$= (5)^3 = \boxed{125}$$

Ex 7 (A) $\sqrt[3]{64 s^9 t^{15}} = \boxed{4s^3t^5}$

$444 \quad s^3s^3s^3 \quad t^5t^5t^5$

OR $64^{\frac{1}{3}}(s^9)^{\frac{1}{3}}(t^{15})^{\frac{1}{3}} = 4s^{\frac{9}{3}}t^{\frac{15}{3}}$

$$= \boxed{4s^3t^5} \checkmark$$

Ex 8 Pg 699 $\sqrt{r^7 s^{25} t^3} = \boxed{r^3 s^{12} t\sqrt{rst}}$

$r^6 \underline{\underline{r}} \quad s^{24} \underline{\underline{s}} \quad t^2 \underline{\underline{t}}$

NOT E1E100 — NO! BOOK SAYS YES

* Solve Ex 9

$$734 = x^{\frac{3}{4}} + 5$$

$$729 = x^{\frac{3}{4}}$$ Raise each side to to $\frac{4}{3}$ power, why?

$$729^{\frac{4}{3}} = \left(x^{\frac{3}{4}}\right)^{\frac{4}{3}}$$

$\boxed{729^{\frac{4}{3}} = x}$ NO CALCULATOR $\quad \therefore x = \left(\sqrt[3]{729}\right)^4 = \boxed{6561}$ CALCULATOR

Homework: Pg 700 #4-17 · Pg 701 # 65-66