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EXPONENTIAL FUNCTION

A function where the independent variable (x) is the exponent

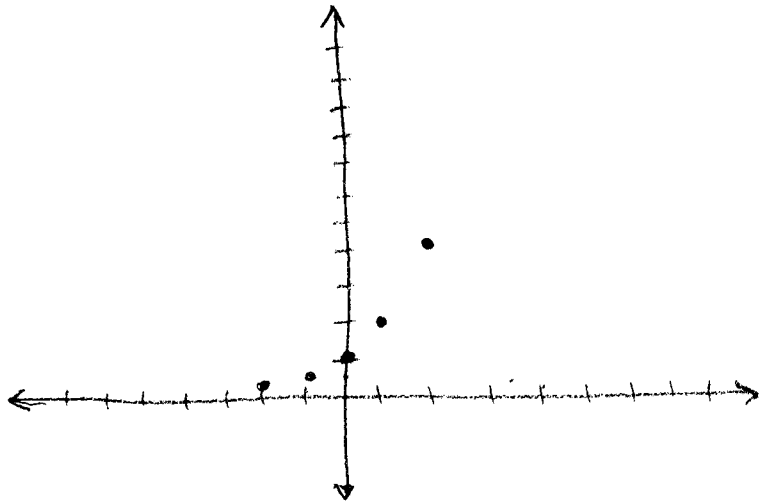
Simple example:  $y = f(x) = 2^x$

GRAPH ANY UNKNOWN FUNCTION... how?

T-Table

X	$2^x$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

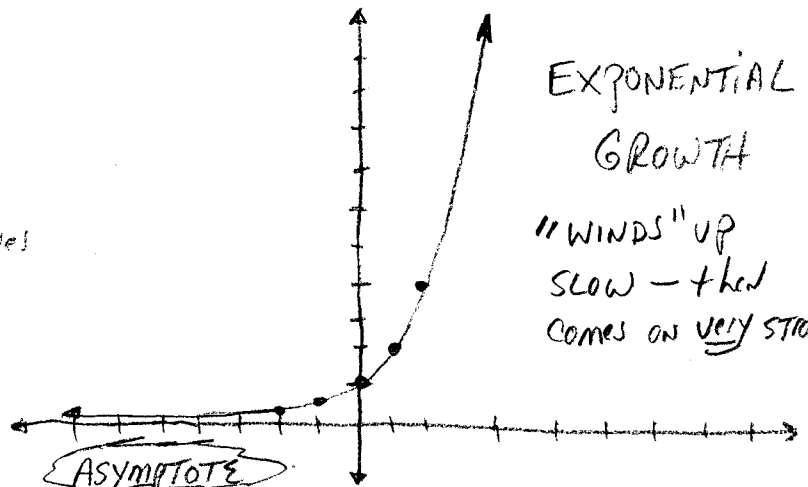
Y-intercept



↑ domain {all TR}  
 ↑ range {>0}

WHAT HAPPENS AS  $x \rightarrow +\infty$ ?  
 AS  $x \rightarrow -\infty$ ?

X	$2^x$
-5	$2^{-5} = \frac{1}{2^5} = \frac{1}{32}$
-Big Number	$\frac{1}{\text{Big Number}} \Rightarrow$ Approaches zero
5	$2^5 = 32$
+ Big Number	$2^{\text{Big Number}} \Rightarrow$ Approaches $\infty$
	$2^{64} = 1.8 \times 10^{19} \Rightarrow$ KING w/ GRAIN OF RICE



EXPONENTIAL GROWTH

"WINDS" UP SLOW - then COMES ON VERY STRONG

In the exponential function, if the base  $y = (\text{base})^x$  is  $< 0$  (negative), then the function is "misbehaved" since the sign of  $y$  will "flip flop" depending on whether or not  $x$  is even ( $+y$ ) or  $x$  is odd ( $-y$ ). So the base must be  $> 0$ .

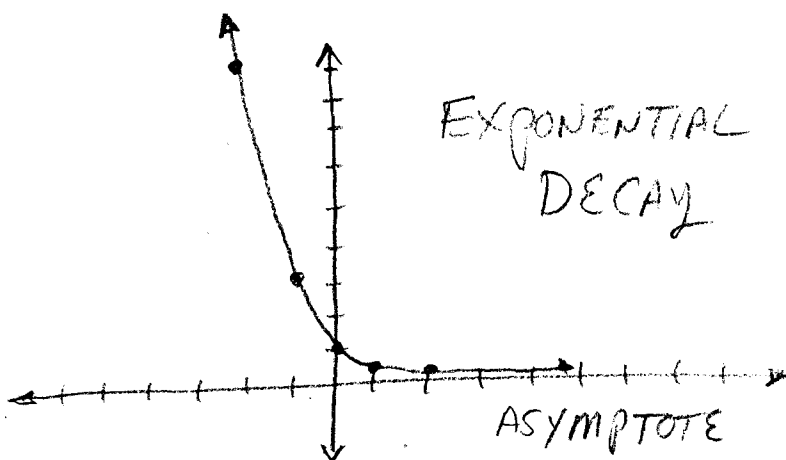
Two other "restrictions" on base:

① What if base is 1  $\Rightarrow y = f(x) = 1^x$   
 $\therefore y$  is always 1  $\Rightarrow$  horizontal line at  $y=1$ .

② What if  $0 < \text{base} < 1$   
 i.e., what if the base is a  $\oplus$  fraction  $< 1$ ?

③  $y = f(x) = \left(\frac{1}{3}\right)^x$

$x$	$\left(\frac{1}{3}\right)^x$
-2	$\frac{1}{\left(\frac{1}{3}\right)^2} = \frac{1}{\frac{1}{9}} = 9$
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$



# GENERAL EXPONENTIAL FUNCTION

$$y = b(1 \pm r)^x$$

independent variable  
⇒ Number of changes

↑  
Y-intercept,  
the starting  
value when X=0  
(0, b)

↑  
RATE OF  
CHANGE } % as a decimal or fraction

Exponential

NOTE: + ⇒ growth (>1)  
- ⇒ decay (<1)

EX)  $y = 2^x$

$$y = 1(1+1)^x$$

↑                    ↑  
b=1                    100% ↑ = 1.00 EACH change

$$y = \left(\frac{1}{3}\right)^x$$

$$y = 1\left(1 - \frac{2}{3}\right)^x \quad \text{or} \quad 1(1 - .\bar{6})^x$$

↳ if you are multiplying by  $\frac{1}{3}$   
Each change, you are "losing"  $\frac{2}{3}$   
each change.

The  $y$  and  $b$  terms ARE NOT common.

Since  $y$  is the "amount" of something you have after " $x$ " changes,  $A$  is often used as in  $A = b(1 \pm r)^x$

To be consistent,  $A_0$ , read "A sub zero" or more often "A naught" is used for the starting (y-intercept) value (when  $x = 0$ )

$$A = A_0 (1 \pm r)^x$$

$\uparrow$                        $\uparrow$   
 ENDING                  STARTING  
 VALUE                      VALUE

EX) The school population increases 2% per year AT SUSAN MOORE - WRITE THE EXPONENTIAL GROWTH function ASSUMING 2009 IS YEAR ZERO AND 600 STUDENTS.

$$A = 600(1.02)^t$$

$A \Rightarrow$  number of students  
 $t \Rightarrow$  years, 2009 = 0

Money has its own language and conventions,

The interest rate is Always per year and is expressed as 2%, 5%, etc. The per year is understood!

To find the "x" or number of changes, you have to know how many times per year the interest is paid, this is called how many times it is compounded

Semi-ANNUAL  $\Rightarrow$  2 per year

quarterly  $\Rightarrow$  4 per year

daily  $\Rightarrow$  365 per year

MONTHLY  $\Rightarrow$  12 per year ... ETC

(EX) Find amount you have if you invest \$5000 for 2 years at 3% compounded quarterly.

RATE  $r = \frac{3\%}{4} = \frac{.03}{4} = .0075$   $r$  is  $\oplus$  for INVESTMENT

NUMBER OF CHANGES  $X = t = (4 \text{ quarters})(2 \text{ years}) = 8 \text{ "changes"}$

$A = 5000(1.0075)^8 = 5000(1.0615988)$

$A = 5307.99$

Money  $\Rightarrow$  Round to hundredths. Why?  
 $A_0 = P = \text{PRINCIPAL}$

# Compare "Simple" vs "Compound" interest

↑  
INTEREST PAID ONCE

↑  
INTEREST PAID OFTEN AND EARNS "interest" ON "interest"

5000 AT 3% simple per year  $\Rightarrow \frac{3}{100} \cdot 5000 = 150$

$\therefore$   $\boxed{\$300 \text{ after 2 years}}$

5000 AT 3% compounded QUARTILY  $\Rightarrow \boxed{307.99 \text{ after 2 years}}$

Exponential functions true a while to "wind up"

Try 50 years  $\Rightarrow$  Simple  $\Rightarrow 150(50) = \boxed{\$7500}$

compound  $\Rightarrow$   $\boxed{\$17283}$   
Use  $A = 5000(1.0075)^{4 \cdot 50}$

• Homework Pg 563 # 6-10

NOTE: depreciation  $\Rightarrow$  decay  $\Rightarrow$  use  $\boxed{-r}$

Summary:  $A = A_0(1 \pm r)^x$   
↑ ENDING AMOUNT    ↑ BEGINNING AMOUNT    ↑ RATE of CHANGE (% as decimal)    Number of changes  
↑ NEGATIVE