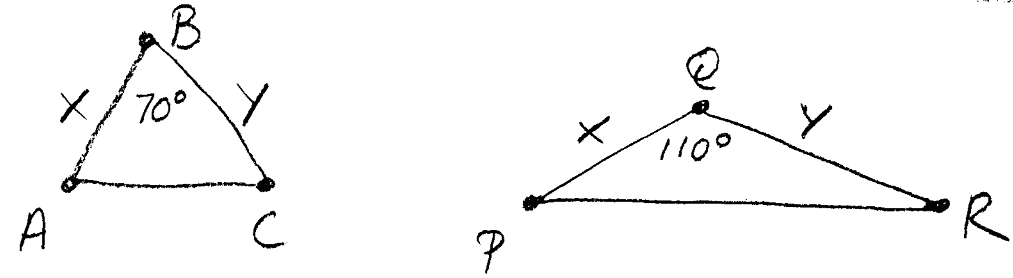
Alg. 2 – BE   Monday 1-24-11

ACT ① $\triangle ABC$ and $\triangle PQR$ are shown below. The given side lengths are in centimeters. The Area of $\triangle ABC$ is 30 square centimeters. What is the area of $\triangle PQR$ in square centimeters?



$$A_{\triangle ABC} = 30 = XY \sin 70 \qquad A_{\triangle PQR} = XY \sin 110°$$

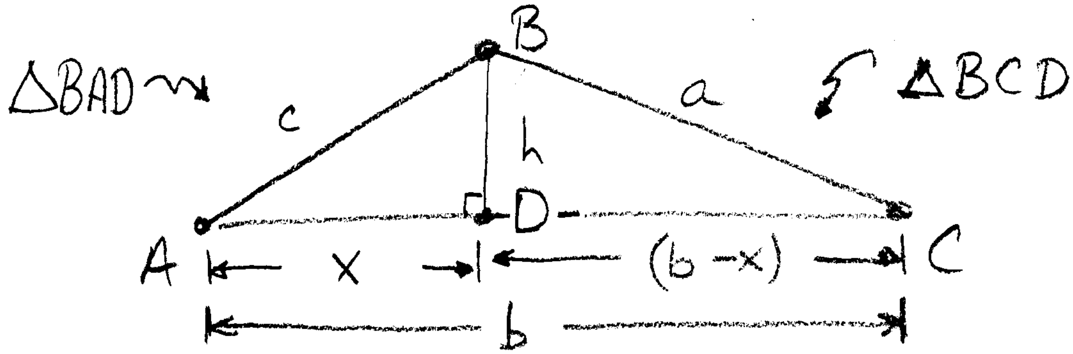
$$\frac{30}{\sin 70} = XY \qquad \therefore A = \frac{30}{\sin 70°} \sin 110°$$

$$\theta' = 70°$$

$$A = \frac{30}{\cancel{\sin} 70} \cancel{\sin} 70$$

$$\boxed{A = 30\,cm^2}$$

LETS LOOK AT ANOTHER $\triangle$ WITH "STANDARD" labels and lets include h. Since we don't know how h (the altitude or height) divides side b, we will label them x and $b-x$ and see where that leads:



$$a^2 = (b-x)^2 + h^2 \qquad \text{P.T. } \triangle BCD$$

$$a^2 = \underbrace{b^2 - 2bx + x^2}_{} + h^2 \qquad \text{"FOIL"}$$

$$c^2 = x^2 + h^2 \qquad \text{P.T. } \triangle BAD$$

$$a^2 = b^2 - 2bx + c^2 \qquad \text{Substitution}$$

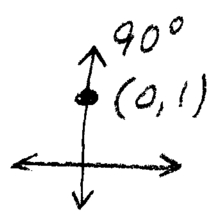
$$\cos A = \frac{x}{c} \quad \therefore \quad x = c\cos A \qquad \text{Substitution}$$

$$a^2 = b^2 - 2b(c\cos A) + c^2$$

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A} \qquad \text{LAW OF COSINES}$$

SAME ENDS

$\underbrace{\qquad\qquad}_{\text{P.T.}}$ $\underbrace{\qquad\qquad}_{\text{WHAT IS } \cos 90°\text{?}}$



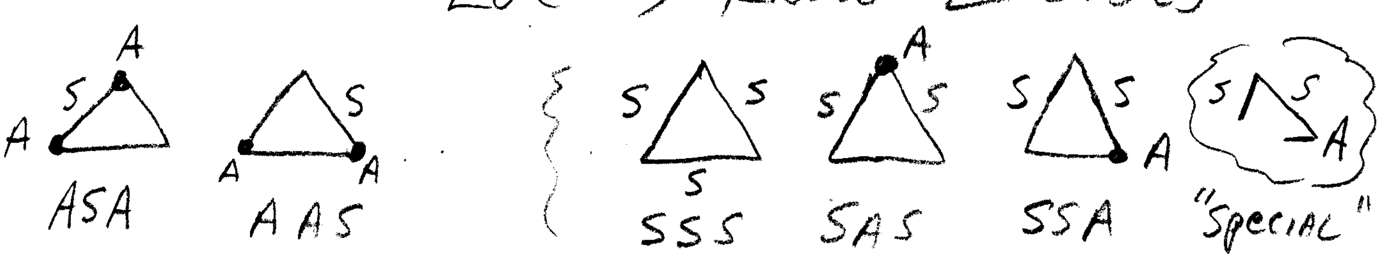
Ch. 13-5 Law of Cosines

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

"3 out of 4" problems
Best if you know 2 sides
eg. SSS, SAS, or *SSA
Special case if angle is opposite short side

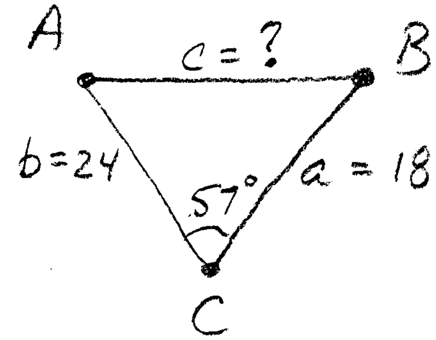
For ASA or AAS use LOS.

For any $\triangle$, you have seen that in Right $\triangle$'s, the sin, cos, tan and P.T. are all you need. But for non-right $\triangle$'s think LOS $\Rightarrow$ Know 2 angles
LOC $\Rightarrow$ Know 2 sides



EX1
Pg 734

Solve $\triangle ABC$



{ SAS ⇒ LOC ⇒ NO WORRIES MATE }

Know a, b, C, perfect for $c^2 =$ {3 KNOWNS} form

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

$$c^2 = 18^2 + 24^2 - 2(18)(24)(\cos 57°)$$

$$c^2 = 324 + 576 - 864(.5446)$$

$$c^2 = 900 - 470.53 = 429.47$$

$$c = \sqrt{429.47} \cong \boxed{20.7 = c}$$

Use LOS to find A or B, lets find A

$$\frac{\sin A}{18} = \frac{\sin 57}{20.7} \quad \therefore \sin A = 18\left(\frac{\sin 57}{20.7}\right)$$

$$\sin A = 18\left(\frac{.8381}{20.7}\right)$$

$$\sin A = .7293$$

$$A = \sin^{-1}(.7293)$$

$$\boxed{A \doteq 46.8°}$$

$$\therefore B = 180 - 46.8 - 57$$

$$\boxed{B = 76.2°}$$

Memory Aid for 2 Angles ⇒ LOS
2 sides ⇒ LOC

LOS ⇒ full of Angles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LOC ⇒ full of sides

$$c^2 = a^2 + b^2 - 2ab\cos C$$

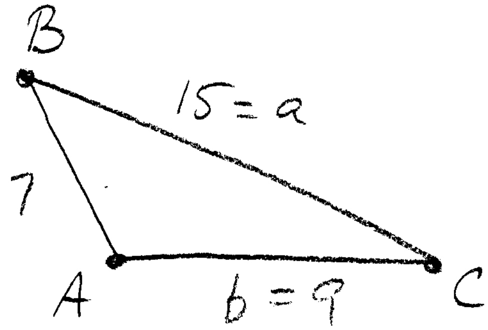
AAS, ASA } LOS

SSS, SAS } LOC

SSA } use LOC ⇒ solving the quadratic will take care of special case ⇒ 2, 1, or 0 positive, real solutions

Ex 2
Pg 734

Solve $\triangle ABC$



SSS

Pick any of the 3 Law of Cosines ⇒
TIP ⇒ Find the largest angle first ⇒

use: $$a^2 = b^2 + c^2 - 2bc\cos A$$

$$15^2 = 9^2 + 7^2 - 2(9)(7)\cos A$$

$$225 = 81 + 49 - 126\cos A$$

$$225 = 130 - 126\cos A$$

$$\frac{95}{-126} = \frac{-126\cos A}{-126}$$

$$-.7540 = \cos A$$

S A
T C
COS NEG. IN Q II
$\theta' = 41.1°$
$\therefore \theta = 180 - 41.1 = 138.9$

$\therefore A = \cos^{-1}(-.7540)$

$\boxed{A \doteq 138.9°}$

Use LOS to find B or C:

$$\frac{\sin B}{9} = \frac{\sin 138.9°}{15} \quad \therefore \sin B = 9\left[\frac{\sin 138.9°}{15}\right]$$

$$\sin B = 9\left[\frac{.6574}{15}\right]$$

$$\sin B = .3944$$

$$B = \sin^{-1}(.3944)$$

$$\boxed{B = 23.2°}$$

$\therefore C = 180 - 138.9 - 23.2 = \boxed{17.9° = C}$

LOOK

HW = Pg 736 #4, 11, 17.