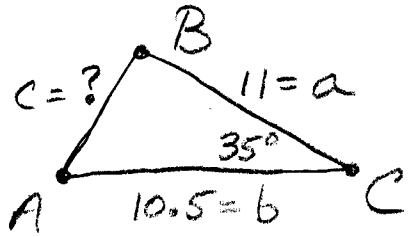
BE - Alg 2 Tuesday 1-25-11

* Homework review
Pg 736 # 4, 11, 17

Homework Review - Pg 736 # 4, 11, 17

(4.) 

SAS ⇒ 2 SIDES ⇒ LofC

Find c FIRST, sides to nearest tenth, ANGLES to nearest degree.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 11^2 + 10.5^2 - 2(11)(10.5)\cos 35^\circ$$

$$c^2 = 121 + 110.25 - 231(.8192)$$

$$c^2 = 231.25 - 189.24$$

$$c^2 = 42.01$$

$$c = \sqrt{42.01} \doteq 6.48 = \boxed{6.5 = c}$$

Find A ⇒ $\frac{\sin A}{11} = \frac{\sin 35^\circ}{6.48}$ ∴ $\sin A = 11\left(\frac{\sin 35^\circ}{6.48}\right)$

DO NOT USE THE ROUNDED 6.5!

$\sin A = 11\left(\frac{.5736}{6.48}\right)$

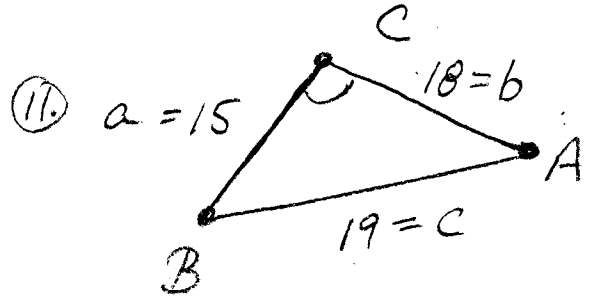
$\sin A = 11(.0885)$

$\sin A = .9735$

$A = \sin^{-1}(.9735)$

$A = 76.8 : \boxed{77^\circ A}$

∴ $B = 180 - 77 - 35 = \boxed{68^\circ = B}$



(11.) $a = 15$

LOOK

* Tip: find biggest Angle ⇒ angle C first ⇒

$$c^2 = a^2 + b^2 - 2ab\cos C$$ ← Get "By itself"

$$19^2 = 15^2 + 18^2 - 2(15)(18)\cos C$$

$$361 = 225 + 324 - 540\cos C$$

$$361 = 549 - 540\cos C$$

$$-549 \quad -549$$

$$\frac{-188}{-540} = \frac{-540\cos C}{-540}$$

$$.3481 = \cos C \quad \therefore C = \cos^{-1}(.3481) = 69.6°$$

$$\therefore \boxed{C = 70°}$$

Find middle side's opposite angle B ⇒

$$\frac{\sin B}{18} = \frac{\sin 70}{19} \quad \therefore \sin B = \frac{18\sin 70}{19}$$

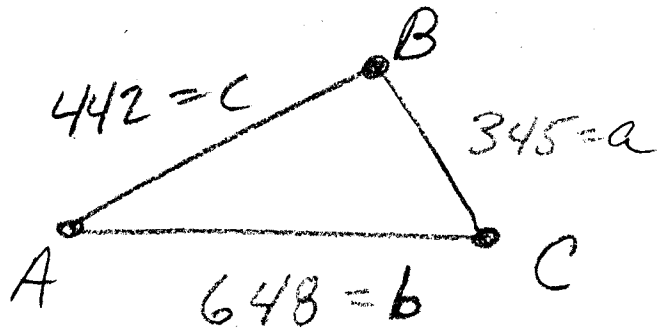
$$\sin B = \frac{18(.9397)}{19} = .8902$$

$$\sin^{-1}(.8902) = \boxed{62.9° = 63° = B}$$

$$\therefore C = 180 - 69.6 - 62.9$$

$$\boxed{C = 47.5 = 48°}$$

(17) $a = 345$
$b = 648$
$c = 442$



LOOK * Find biggest side first

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$648^2 = 345^2 + 442^2 - 2(345)(442)\cos B$$

$$419904 = 119025 + 195364 - 304980\cos B$$

$$419904 = 314389 - 304980\cos B$$

$$105515 = -304980\cos B$$

$$-0.3460 = \cos B \quad \therefore \cos^{-1}(-.3460) = 110.2^\circ$$

$$= \boxed{110^\circ = B}$$

Find middle sides opposite angle $\Rightarrow C$

$$\frac{\sin C}{442} = \frac{\sin 110}{648} \quad \therefore \quad \sin C = 442\frac{(\sin 110)}{648}$$

$$= .6821\,(.9397)$$

$$\sin C = .6410$$

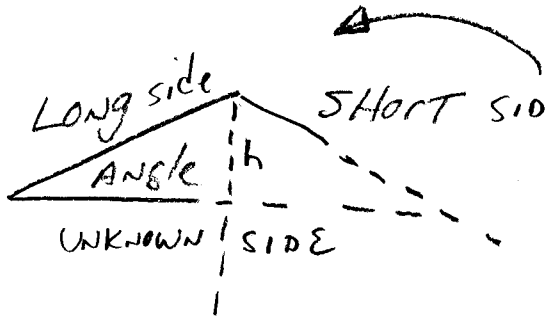
$$C = \sin^{-1}(.6410)$$

$C = 39.9^\circ = \boxed{40^\circ = C}$

$A \Rightarrow 180 - 110.2 - 39.9 = 29.2$

$\boxed{A = 30^\circ}$

If you know two sides And ONE Angle, and the Angle is OPPOSITE THE SHORT side



SSA

There MAY BE NONE, ONE, or two possible SOLUTIONS.

The Law of Cosines RESULTS IN A QUADRATIC WITH 0 positive, 1 positive, or 2 positive solutions.

Law of Sines = not recommended for SSA - must memorize the table on next page if you want to use it!! - not recommended.

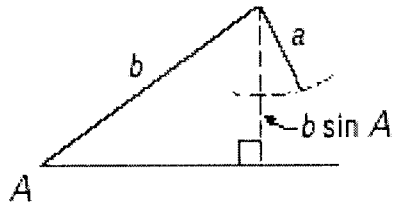
(see 727 & 728 if you want to) use the Law of SINES

* called the Ambiguous Case of the LOS

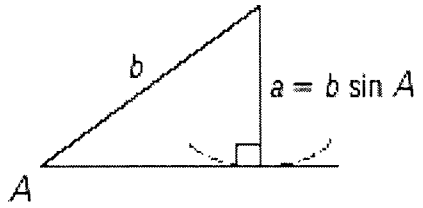
Key Concept — Possible Triangles Given Two Sides and One Opposite Angle

Suppose you are given a, b, and A for a triangle.

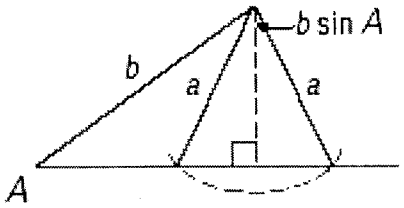
A Is Acute ($A < 90°$).



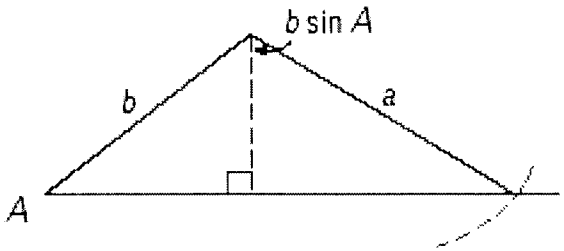
$a < b \sin A$
no solution



$a = b \sin A$
one solution

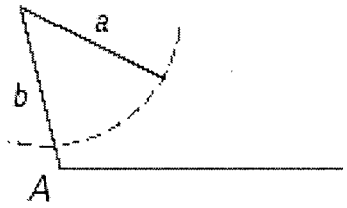


$b > a > b \sin A$
two solutions

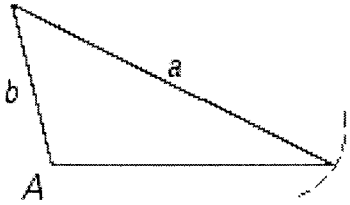


$a \geq b$
one solution

A Is Right or Obtuse ($A \geq 90°$).



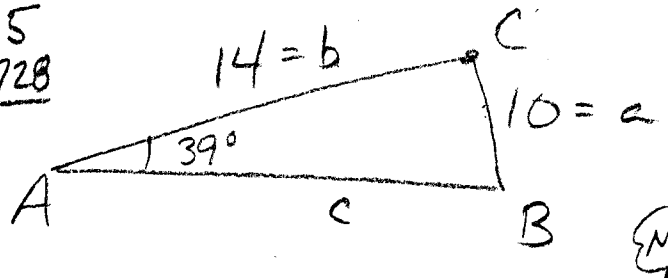
$a \leq b$
no solution



$a > b$
one solution

The Ambiguous Case of the Law of Sines = SSA ==> Use LOC and QE -- Mr.C.

EX 5
Pg 728



SSA
↑↑
TWO SIDES ⇒ LOC

NOLO COS A Form

LOC ⇒ KNOW a, b, COS A ONLY C IS UNKNOWN

⇒ "3 of 4" $a^2 = b^2 + c^2 - 2bc\cos A$

$10^2 = 14^2 + c^2 - 2(14)c(\cos 39)$

$0 = 96 + c^2 - 28(.7771)c$

$c^2 - 21.76c + 96 = 0$

$a = 1$
$b = -21.76$
$c = 96$

$b^2 - 4ac$

$(-21.76)^2 - 4(1)(96)$

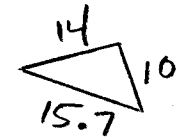
$473.5 - 384 = 89.5 = d$

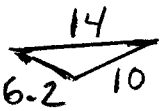
$$c = \frac{-b \pm \sqrt{d}}{2a} = \frac{21.76 \pm \sqrt{89.5}}{2(1)}$$

$$c = \frac{21.76 \pm 9.46}{2}$$

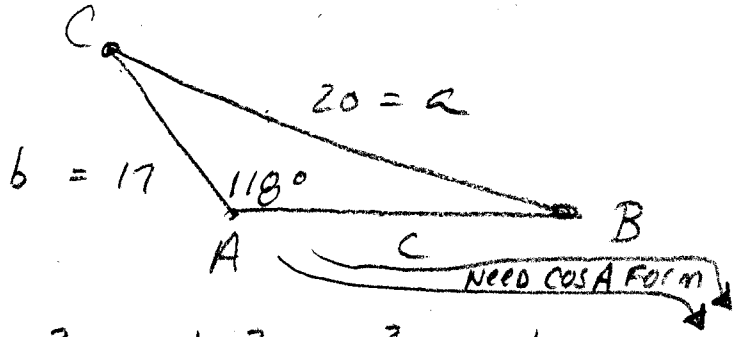
$c \cong \{15.7, 6.2\}$

TWO SOLUTIONS





Ex 3
Pg 727



SSA
Two Sides LOC

$LOC \Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$

$20^2 = 17^2 + c^2 - 2(17)c(\cos 118^\circ)$

$400 = 289 + c^2 - 34(-.4695)c$

$0 = -111 + c^2 + 15.963c$

OR $c^2 + 15.963c - 111 = 0$

$a = 1$

$b = 15.963$

$c = -111$

$b^2 - 4ac$

$(15.963)^2 - 4(1)(-111)$

$254.82 + 444 = 698.82 = d$

$$c = \frac{-b \pm \sqrt{d}}{2a} = \frac{-15.963 \pm \sqrt{698.82}}{2(1)}$$

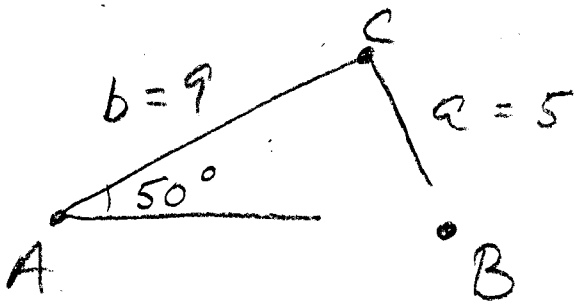
$$c = \frac{-15.963 \pm 26.44}{2}$$

$c = 5.23$ and -21.2 discard

$c = 5.23$

ONE SOLUTION

Ex 4
Pg 728



KNOWN Angle OPPOSITE SHORTSIDE

{SSA}

2, 1, or 0 solutions

Know b, a, $\cos 50 = \cos A$ → Need $\cos A$ Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$5^2 = 9^2 + c^2 - 2(9)c(\cos 50)$$

$$0 = 56 + c^2 - 18(.6428)c$$

$$0 = c^2 - 11.57c + 56$$

$a = 1$

$b = -11.57$

$c = 56$

$$b^2 - 4ac$$

$$(-11.57)^2 - 4(1)(56)$$

$$133.87 - 224 = -90.13 = d$$

↑

d is negative

∴ [No Real Solutions]

Summary: ASA, AAS } Law of Sines

SAS } Law of Cosines

SSS } Law of Cosines, solve for largest angle first

*SSA } Law of Cosines ⇒ Quadratic Equation w/ 0, 1, 2 solutions.

*(IF KNOW ANGLE IS OPPOSITE SHORT SIDE)

- Homework: Pg 736 # 12, 13, 14

USE: LotS, LotC, LotC

↓

QUADRATICS. SOLVE BOTH Δ