BE-Alg. 21 Tuesday 4-5-11

Simplify each expression, name the exponent rule used.

① $x^{12} \cdot x^{7} \cdot x^{5}$

② $\dfrac{x^{18}}{x^{5}}$

③ $(x^{4})^{9}$

Evaluate:

④ $\log_3 243$

Solve:

⑤ $\log_8 x = \dfrac{4}{3}$

ANS

① x^{24} ⇒ Mult. Rule ⇒ add exp.

② x^{13} ⇒ Division Rule ⇒ subt. exp.

③ x^{36} ⇒ Power-Power Rule ⇒ mult exp.

What is a logarithm? ↷ Keep this in mind for later

④ $3^{x} = 343$ $\boxed{x = 5}$

⑤ $8^{\frac{4}{3}} = x$ ∴ $\left(\sqrt[3]{8}\right)^{4} = \boxed{16}$

Ck $\log_8 16 \stackrel{?}{=} \frac{4}{3}$, $8^{\frac{4}{3}} \stackrel{?}{=} 16$ ✓

Alg. 2 Homework Review #4-13, Pg. 535

④ $5^4 = 625$ ∴ $\boxed{\log_5 625 = 4}$

⑤ $7^{-2} = \frac{1}{49}$ ∴ $\boxed{\log_7 \left(\frac{1}{49}\right) = -2}$

⑥ $\log_3 81 = 4$ ∴ $\boxed{3^4 = 81}$

⑦ $\log_{36} 6 = \frac{1}{2}$ ∴ $\boxed{36^{\frac{1}{2}} = 6}$

⑧ $\log_4 256 \Rightarrow 4^x = 256$ ∴ $x = 4$

∴ $\boxed{\log_4 256 = 4}$

⑨ $\log_2 \frac{1}{8} \Rightarrow 2^x = \frac{1}{8}$ ∴ $x = -3$

∴ $\boxed{\log_2 \frac{1}{8} = -3}$

⑩ $3^{\log_3 21} \Rightarrow$ $\log_3 21$ is AN EXPONENT, it is the EXPONENT of 3 that gives you 21 ∴ $3^{\log_3 21} = \boxed{21}$

See Ex 4 Pg 533 ④b $3^{\log_3(4x-1)} = 4x-1$

$\boxed{\text{GENERAL: } b^{\log_b x} = x}$ Logic

(11) $\log_5 5^{-1} = \boxed{-1}$

↳ 5 raised to what power would equal 5^{-1}??

See Ex 4 Pg 533

(4A) $\log_6 6^8 = 8$

General: $\log_b b^x = x$

(12) $\log_9 x = \frac{3}{2}$ $\therefore 9^{\frac{3}{2}} = x$

$\sqrt[2]{9^3}$ or $\left(\sqrt[2]{9}\right)^3 = \boxed{27}$

CK $\log_9 27 \stackrel{?}{=} \frac{3}{2}$

$9^{\frac{3}{2}} \stackrel{?}{=} 27$ ✓

(13) $\log_{\frac{1}{10}} x = -3$ $\therefore \left(\frac{1}{10}\right)^{-3} = x$

$\frac{1}{\left(\frac{1}{10}\right)^3} = x$

$\frac{1}{\frac{1}{1000}} = x$

$\boxed{1000 = x}$

CK $\log_{\frac{1}{10}} 1000 \stackrel{?}{=} -3$

$\left(\frac{1}{10}\right)^{-3} \stackrel{?}{=} 1000$

$\frac{1}{\left(\frac{1}{10}\right)^3} = \frac{1}{\frac{1}{1000}} = 1000$ ✓

The logarithmic function

$$y = \log_b x$$

Requires b, the base: $b > 0$, $b \neq 1$

and $x > 0$ Why $x > 0$?

Try $y = \log_5 125$

$125 = 5^y$ $\therefore y = 3$ OK.

Try $y = \log_5 (-125)$

$-125 = 5^y$

How is raising 5 to any y value going to change the positive base to a negative.

If cannot, $\therefore x > 0$ and $b > 0$, $b \neq 1$

Checking inverses:

Let $\boxed{y = f(x) = b^x}$ EXPONENTIAL FUNCTION

$f^{-1}(x) \Rightarrow \quad x = b^y$

$\Rightarrow \quad \log_b x = y$

Let $\boxed{y = g(x) = \log_b x}$ logarithmic function

Show $f[g(x)] = g[f(x)] = x$

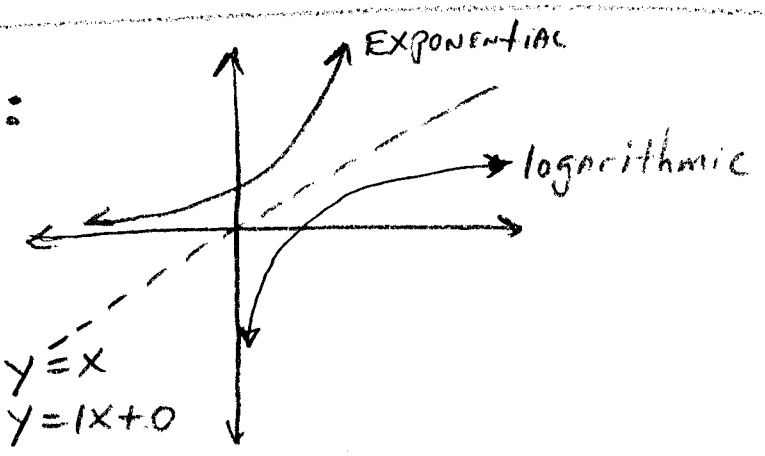
$f(x) = b^x$

$f(\log_b x) = b^{\log_b x} = x$ ✓

$g(x) = \log_b x$

$g(b^x) = \log_b b^x = x$ ✓

GENERAL:



Suppose: $\log_3 X = \log_3 81$

What must be true?

Recall: • A logarithm is an exponent.
• All exponents have bases

$$\log_3 X = \log_3 81$$

The exponent of base 3 that produces X = the exponent of base 3 that produces 81

$\therefore X = 81$

Ex 7 pg 534 Solve $\log_5 (p^2 - 2) = \log_5 p$

$\therefore p^2 - 2 = p$

$p^2 - p - 2 = 0$

Sum $\Rightarrow -1$
Prod $\Rightarrow -2$
$+1 \quad -2$

$(p+1)(p-2) = 0 \qquad p = \{-1, 2\}$

CK $\log_5 ((-1)^2 - 2) = \log_5 (-1)$
UNDEFINED $\therefore$ EXTRANEOUS SOLUTION

CK $\log_5 ((2)^2 - 2) = \log_5 2$ ✓

Pg. 535 # 18-20

$$L = 10 \log_{10} R$$

L: Loudness, in decibels

R: Relative intensity of sound

(18) Solve $\frac{130}{10} = \frac{10 \log_{10} R}{10}$ for R

$$13 = \log_{10} R$$

$$\boxed{10^{13} = R}$$ Fireworks

(19) Solve $7.5 = 10 \log_{10} R$ for R

$$7.5 = \log_{10} R$$

$$\boxed{10^{7.5} = R}$$ concert between 10^7 and 10^8

(20) Ratio? $\frac{10^{13}}{10^{7.5}} = \boxed{10^{5.5}}$

$10^5 < 10^{5.5} < 10^6$

$100{,}000 < x < 1{,}000{,}000$

$x \approx 316{,}228$

Homework: Pg 536 # 21-24, 30-32, 39, 40, 47, 50, 54.