

BE-Alg. 2 | Thursday 5-5-11

From memory:

- ① List THE 3 logarithm rules
 - ② List e to 10 digits.
-

The exponential growth or decay function can be broken down into its more general parts.

$$y = a(1 \pm r)^x$$

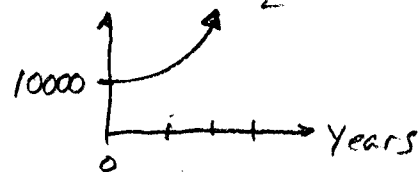
↑ ↑ ↑ ↑
 AMOUNT "STARTING VALUE" ⇒ WHAT YOU HAVE WHEN $x=0$ CHANGE AMOUNT (AS A DECIMAL) number of changes

Population increases 8% per year starting in 1985 when $P=10000$

$$P = P_0(1 + .08)^t \quad t=0=1985$$

↑
the "sub zero" indicates starting value

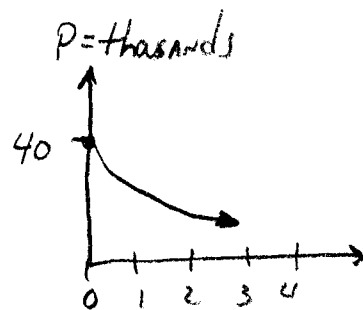
$$P = 10000(1.08)^t$$



Population decrease 5% per year starting in 2000 when $P=40000$

$$P = 40000(1 - .05)^t \quad t=0=2000$$

$$P = 40000(.95)^t$$



Ch. 10-6 Exponential Growth AND Decay

Ex Swine flu cases, $\uparrow 10\%$ per month,
12000 in Jan. 2008

$$F = F_0 (1 + .10)^m \quad m=0 = \text{Jan. 2008}$$

$$F = 12000 (1.10)^m$$

A Find Number of cases in Oct. 2009

$$m = 22 \therefore F = 12000 (1.10)^{22}$$

$$F = 97683$$

Ex Car depreciates 15% per year. Bought new
in 2002 for 18000. Present value?

$$C = C_0 (1 - .15)^t \quad t=0 = 2002$$

$$\text{Model: } C = 18000 (.85)^t \quad t=8 \text{ in 2010}$$

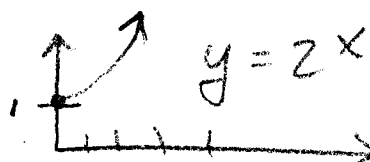
$$\therefore C = 18000 (.85)^8$$

$$C = 4904.83$$

Ex Something $\uparrow 100\%$ per period X. Starting value = 1

$$Y = 1 (1 + 1.00)^x$$

$$Y = 2^x$$



Money has its own vocabulary and

CONVENTIONS: Rate $\Rightarrow$ is Assumed to be per year.

P = Principal = Starting Amount

N = Compounding period
(how many times per year interest is paid)

(Ex) \$5000 invested AT 6% compounded quarterly

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 Principal growth per year 4 times per yr

$$A = 5000 \left(1 + \frac{.06}{4}\right)^{4 \cdot t}$$

$t = 0 = \text{now}$
 t is in years

↑
interest rate
per quarter

NUMBER OF
QUARTERS

$$A = 5000 (1.015)^{4t}$$

Find A in 10 years

$$A = 5000 (1.015)^{4 \cdot 10}$$

$$A = 9070.09$$

General: $A = P \left(1 \pm \frac{r}{N}\right)^{Nt}$ Compound Interest Formula

WHAT HAPPENS AS N GETS VERY BIG?

Let's look at 8% (per year) for various compounding periods and compare $(1+r)^t$ and e^{rt} .

Assume $P=1$
AND $t=1$
 N = compounding period

$$\therefore A = 1\left(1+\frac{r}{N}\right)^{Nt} \quad \text{or} \quad A = 1e^{rt}$$

N	$\left(1+\frac{r}{N}\right)^{N \cdot 1}$	$e^{r \cdot 1}$
1	1.08	1.083287
4	1.082432	1.083287
12	1.083	1.083287
365	1.083278	1.083287
730	1.083282	1.083287
1095	1.083284	1.083287
3650	1.083286	1.083287



As N approaches

"continuously" compounded, the result approaches base "e" $\Rightarrow$ Why? Because:

$$\lim_{N \rightarrow \infty} \left(1 + \frac{1}{N}\right)^N = e$$

e is the "NATURAL base" for continuous changes

Two forms of Exponential Growth AND Decay:

Non-Continuous
Ex) Monthly, yearly, ...

$$y = a(1 \pm r)^t$$

Continuous

$$y = ae^{\pm rt}$$

or $y = ae^{\pm kt}$

Homework: • Read Ch. 10-6

• Pg 563 #10, 11.