

Alg. 2 BE | Friday 5-6-11

- ① Which model is for "continuous" exponential growth/decay and which is for "non-continuous"?

$$\boxed{y = a(1 \pm r)^t} \text{ or } \boxed{y = ae^{\pm rt}}$$

- ② Which models each problem best, continuous or non-continuous exponential change?
- Ⓐ Investment of 5% compounded quarterly.
 - Ⓑ Radioactive decay.
 - Ⓒ Population change based on annual census.
 - Ⓓ Investment of 5% compounded every minute.
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Alg. 2 Homework Review - Pg. 563 #10, 11

- ⑩ Computer = \$2500 Depreciates 20% per year
Value in 2 years? $\downarrow$ = 0.20 per year

$$C = C_0 (1 \pm r)^t$$

$$C = 2500(1 - 0.20)^2 = 2500(.8)^2 = 2500(.64)$$

$$\boxed{C = \$1600} = \text{value of computer after 2 years}$$

- ⑪ Condo = \$85,000 Value $\uparrow$ 5% per year
Value in 5 years = .05 per year

$$V = V_0 (1 \pm r)^t$$

$$V = 85000(1 + 0.05)^5$$

$$V = 85000(1.05)^5 = 85000(1.2763...)$$

$$\boxed{V = \$108,484}$$

SOLVING EXPONENTIAL GROWTH/decay EQUATIONS

EX1
Pg 560

Cup of coffee = 130 mg caffeine

↓ Eliminated (body) $\Rightarrow$ 11% = .11 per hr.

How long to eliminate half?

C = mg of caffeine

$$C = C_0 (1 - r)^t$$

$t=0$ = drank 1 cup of coffee

$$C = 130 (1 - .11)^t$$

To eliminate $\frac{1}{2}$

$$\Rightarrow \frac{130}{2} = 65 \text{ mg}$$

$$65 = 130 (.89)^t$$

Solve for t

$$\frac{65}{130} = (.89)^t$$

$$\frac{1}{2} = .89^t$$

Take log or ln of both sides

$$\ln\left(\frac{1}{2}\right) = \ln(.89)^t$$

$$\ln\left(\frac{1}{2}\right) = t \ln(.89)$$

$$\frac{\ln\left(\frac{1}{2}\right)}{\ln(.89)} = t$$

$$\ln(.89)$$

$$\therefore t = \frac{- .693147}{- .116534} = \boxed{5.9480 \text{ hours}}$$

EX2 RADIOACTIVE Decay
Pg 561 $\Rightarrow$ use $A = A_0 e^{-kt}$

① Half-life $\Rightarrow$ time it takes for $\frac{1}{2}$ the atoms in a substance to decay.

$$T_{1/2} \text{ of C-14} = 5760 \text{ years}$$

Find k for Carbon-14

$$A = A_0 e^{-kt} \quad \text{and} \quad \frac{A}{A_0} = \frac{1}{2} \quad \text{if } t = 5760 \text{ years}$$

$$\therefore \frac{1}{2} = e^{-k(5760)}$$

$$\ln\left(\frac{1}{2}\right) = \ln e^{-5760k}$$

$$\frac{-0.693147}{-5760} = \frac{-5760k}{-5760}$$

$$\therefore k = -1.2034 \times 10^{-4}$$

$$k = -0.00012034$$

$\therefore$ Model the decay of C-14

$$\text{WITH } A = A_0 e^{-0.00012034t} \quad t = \text{years}$$

- ⑥ Use your model for C-14 decay to estimate how long it takes for C-14 to reach 3% of starting value
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$$\Rightarrow \frac{A}{A_0} = .03 \quad \text{and} \quad A = A_0 e^{-.00012034 t}$$

$$\therefore .03 = e^{-.00012034 t}$$

$$\ln(.03) = -.00012034 t$$

$$\frac{\ln(.03)}{-.00012034} = t \quad \text{where } t = \text{years}$$

$$\frac{-3.506557}{-.00012034} = t$$

$$\boxed{29138.76 \text{ years} = t}$$

EX4
Pg 562

China $\Rightarrow 1.26$ billion $= 1.26 \times 10^9$

India $\Rightarrow 1.01$ billion $= 1.01 \times 10^9$

Models $\Rightarrow$

$$C = 1.26e^{.009t}$$

$$I = 1.01e^{.015t}$$

When will $I > C$?

$$1.01e^{.015t} > 1.26e^{.009t}$$

$$\ln(1.01)(e^{.015t}) > \ln(1.26)(e^{.009t})$$

$$\therefore \ln(1.01) + \ln(e^{.015t}) > \ln(1.26) + \ln(e^{.009t})$$

$$0.0099503 + .015t > .231112 + .009t$$

$$-0.0099503 - .009t + 0.0099503 - .009t$$

$$\frac{.006t}{.006} > \frac{.221162}{.006}$$

$$t > 36.86 \text{ years}$$

Homework: Pg 564 #15-20