14-3 **Study Guide**

Trigonometric Identities

Simplify Expressions The simplified form of a trigonometric expression is written as a numerical value or in terms of a single trigonometric function, if possible. Any of the trigonometric identities on page 849 can be used to simplify expressions containing trigonometric functions.

Example 1 Simplify $(1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta$.

$$(1 - \cos^2 \theta) \sec \theta \cot \theta + \tan \theta \sec \theta \cos^2 \theta = \sin^2 \theta \cdot \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\cos \theta} \cdot \cos^2 \theta$$
$$= \sin \theta + \sin \theta$$
$$= 2 \sin \theta$$

Example 2 Simplify $\dfrac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \dfrac{\csc \theta}{1 + \sin \theta}$.

$$\frac{\sec \theta \cdot \cot \theta}{1 - \sin \theta} - \frac{\csc \theta}{1 + \sin \theta} = \frac{\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}}{1 - \sin \theta} - \frac{\frac{1}{\sin \theta}}{1 + \sin \theta}$$

$$= \frac{\frac{1}{\sin \theta}(1 + \sin \theta) - \frac{1}{\sin \theta}(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$= \frac{\frac{1}{\sin \theta} + 1 - \frac{1}{\sin \theta} + 1}{1 - \sin^2 \theta}$$

$$= \frac{2}{\cos^2 \theta}$$

Exercises

Simplify each expression.

1. $\dfrac{\tan \theta \cdot \csc \theta}{\sec \theta}$

2. $\dfrac{\sin \theta \cdot \cot \theta}{\sec^2 \theta - \tan^2 \theta}$

3. $\dfrac{\sin^2 \theta - \cot \theta \cdot \tan \theta}{\cot \theta \cdot \sin \theta}$

4. $\dfrac{\cos \theta}{\sec \theta - \tan \theta}$

5. $\dfrac{\tan \theta \cdot \cos \theta}{\sin \theta} + \cot \theta \cdot \sin \theta \cdot \tan \theta \cdot \csc \theta$

6. $\dfrac{\csc^2 \theta - \cot^2 \theta}{\tan \theta \cdot \cos \theta}$

7. $3 \tan \theta \cdot \cot \theta + 4 \sin \theta \cdot \csc \theta + 2 \cos \theta \cdot \sec \theta$

8. $\dfrac{1 - \cos^2 \theta}{\tan \theta \cdot \sin \theta}$

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Verifying Trigonometric Identities

Verify that each of the following is an identity.

1. $\dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = \sec^2 \theta$

2. $\dfrac{\cos^2 \theta}{1 - \sin^2 \theta} = 1$

3. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

4. $\tan^4 \theta + 2 \tan^2 \theta + 1 = \sec^4 \theta$

5. $\cos^2 \theta \cot^2 \theta = \cot^2 \theta - \cos^2 \theta$

6. $(\sin^2 \theta)(\csc^2 \theta + \sec^2 \theta) = \sec^2 \theta$

Bonus

Textbook Problem: Page 784 #16.