

BE-Geometry 1 | WEDNESDAY 1-5-11

① Name the tools you would use to solve triangles WITH the following known sides or angles:

Ⓐ ASA

Ⓑ SAS

Ⓒ AAS

Ⓓ SSA

Ⓔ SSS

Ⓕ AAA

② From memory, write formulas for:

Ⓐ Pythagorean Theorem Ⓑ Sin, cos, Tan

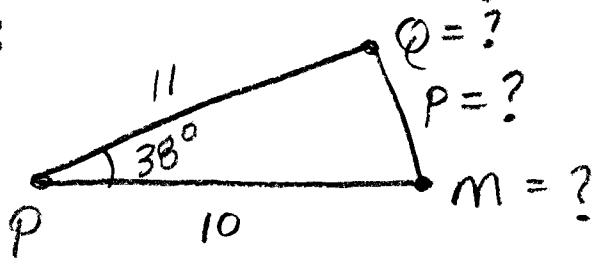
Ⓒ Law of Sines Ⓓ Law of Cosines

• Homework review

Homework Review Pg 388 # 20, 21

SOLVE:

(20)



∠s to nearest degree
sides to nearest tenth

USE THE LOC WITH KNOWN ANGLE P

$$p^2 = 11^2 + 10^2 - 2 \cdot 11 \cdot 10 \cdot \cos 38^\circ$$

$$p^2 = 121 + 100 - 220(.7880)$$

$$p^2 = 221 - 173.36 \quad \therefore p = \sqrt{47.64}$$

$$\boxed{p \approx 6.9}$$

$$\therefore \frac{\sin 38^\circ}{6.9} = \frac{\sin Q}{10}$$

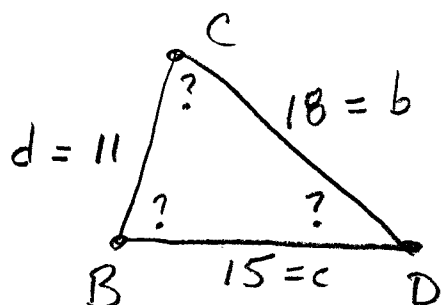
$$Q = \sin^{-1} \left[\frac{10 \sin 38}{6.9} \right] = \sin^{-1} \left[\frac{6.157}{6.9} \right] = \sin^{-1} [.8923]$$

$$\boxed{Q \approx 63^\circ}$$

$$\therefore m = 180 - (38 + 63)$$

$$\boxed{m = 79^\circ}$$

(21)

TIP: USE LARGEST SIDE TO FIND LARGEST $\angle$ FIRST

$$b^2 = d^2 + c^2 - 2dc \cos B$$

$$18^2 = 11^2 + 15^2 - 2(11)(15) \cos B$$

$$-22 = -330 \cos B$$

$$\therefore \cos B = \frac{22}{330} = .0667$$

$$\cos^{-1}(.0667) \approx \boxed{86^\circ = B}$$

$$\frac{\sin 86^\circ}{18} = \frac{\sin D}{11}$$

$$\therefore \sin D = \frac{11 \sin 86^\circ}{18}$$

$$D = \sin^{-1} \left[\frac{11(.9976)}{18} \right]$$

$$D = \sin^{-1} [.6096]$$

$$\boxed{D \approx 38^\circ}$$

$$\therefore C = 180 - (38 + 86)$$

$$\boxed{C = 56^\circ}$$

THE SSA CASE IS DIFFERENT BECAUSE
FOR ANY SSA, THERE MAY BE NO
TRIANGLES, ONE TRIANGLE, OR TWO TRIANGLES
THAT EXIST WITH THAT PARTICULAR SSA.

MOST BOOKS CALL THE THE
"Ambiguous" CASE OF THE LAW OF SINES
(unclear)

AND FORCE YOU TO MEMORIZE THE
CONDITIONS FOR EACH POSSIBILITY.
*(see next page)

BUT THERE IS, IMHO, A BETTER WAY.

YOU CAN USE THE LAW OF COSINES,
SOLVE THE RESULTING QUADRATIC EQUATION
(EASY WITH A CALCULATOR) AND LET THE
ANSWER TELL YOU WHAT YOU NEED TO KNOW.

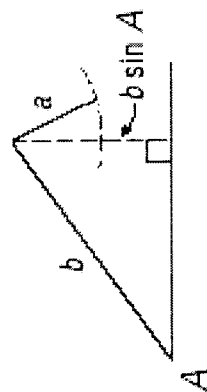
ie., 0, 1, OR 2 SOLUTIONS. HERE'S HOW:

Key Concept

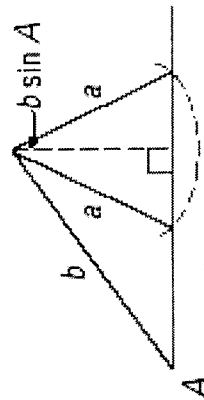
Possible Triangles Given Two Sides and One Opposite Angle

Suppose you are given a , b , and A for a triangle.

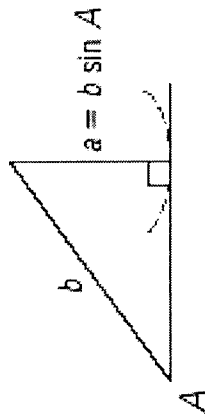
A Is Acute ($A < 90^\circ$).



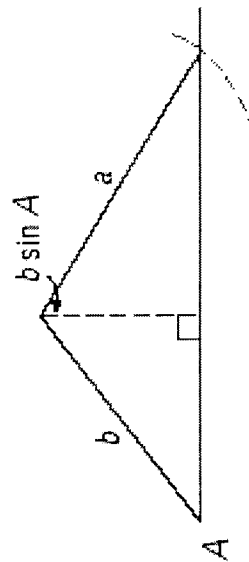
$a < b \sin A$
no solution



$b > a > b \sin A$
two solutions

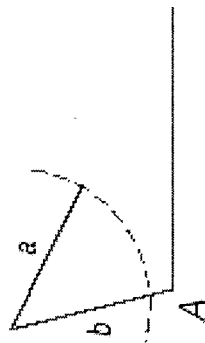


$a = b \sin A$
one solution

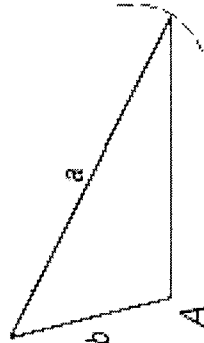


$a \geq b$
one solution

A Is Right or Obtuse ($A \geq 90^\circ$).



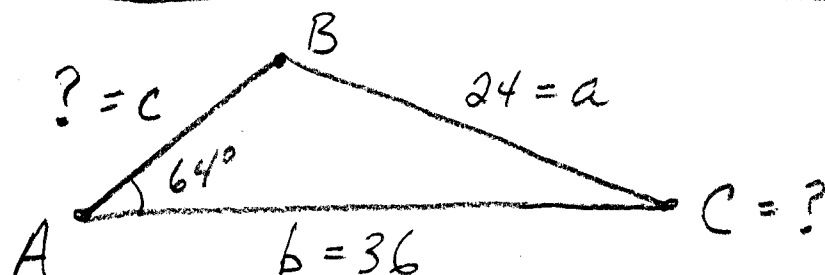
$a \leq b$
no solution



$a > b$
one solution

The Ambiguous Case of the Law of Sines = SSA ==> Use LOC and QE -- Mr.C.

(Ex) Solve: $a=24$ $b=36$ $A=64^\circ$



- SSA $\Rightarrow$ Law of Cosines
- Use the known angle $\Rightarrow A = 64^\circ$

$$24^2 = 36^2 + c^2 - 2 \cdot 36 \cdot c \cdot \cos 64^\circ$$

$$576 - 1296 = c^2 - 72c \cdot (0.4384)$$

$$-720 = c^2 - 31.5648c$$

$$\therefore c^2 - 31.5648c + 720 = 0$$

$$a = 1 \quad b^2 - 4ac$$

$$b = -31.5648 \quad (-31.5648)^2 - 4(1)720$$

$$c = 720$$

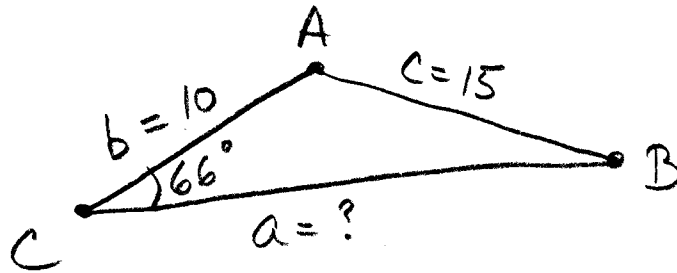
$$996.3366 - 2880 = -1883.6634$$

"discriminant"

Since d is negative, this

triangle has no real solution

⊗ EX $b=10, c=15, C=66^\circ$



$$15^2 = 10^2 + a^2 - 2 \cdot 10 \cdot a \cdot \cos 66^\circ$$

$$125 = a^2 - 20a(0.4067)$$

$$a^2 - 8.134a - 125 = 0$$

$$a = 1$$

$$b^2 - 4ac$$

$$b = -8.134$$

$$(-8.134)^2 - 4(1)(-125)$$

$$c = -125$$

$$66.1620 + 500 = 566.1620 = d$$

$$a = \frac{-b \pm \sqrt{d}}{2a} = \frac{8.134 \pm \sqrt{566.1620}}{2}$$

$$a = \frac{8.134 \pm 23.794}{2}$$

NOT POSSIBLE

$$\therefore \{a = 15.964, -7.83\}$$

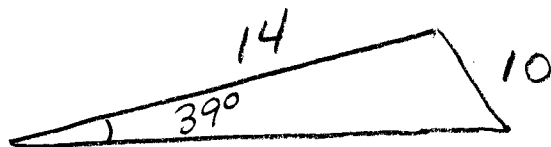
↑
ONLY POSSIBLE ANSWER FOR a

FIND A, use LOS $\Rightarrow \frac{\sin A}{15.964} = \frac{\sin 66^\circ}{15}$

$$A = \sin^{-1} \left[\frac{15.964 \cdot \sin 66^\circ}{15} \right] = \sin^{-1} [0.9722] \therefore A = 76.5^\circ$$

$$\therefore B = 180 - [66 + 76.5] = 37.5^\circ = B$$

(EX)



* SSA, $\Rightarrow$ LOC $\Rightarrow$ 0, 1, or 2 solutions possible

Must use } $(\text{KNOWN SIDE})^2 = (\quad)^2 + (\quad)^2 - 2(\quad)(\quad)\cos(\text{KNOWN angle})$
 LOC Form }
 LIKE THIS } $\rightarrow$ OPPOSITE $\leftarrow$

$$10^2 = 14^2 + X^2 - 2(14)(X)\cos 39^\circ$$

$$100 = 196 + X^2 - 28X(.7771)$$

$$0 = 96 + X^2 - 21.7588X$$

QUADRATIC!
 USE QF

$$X^2 - 21.7588X + 96 = 0$$

$$a = 1$$

$$b = -21.7588$$

$$c = 96$$

$$b^2 - 4ac$$

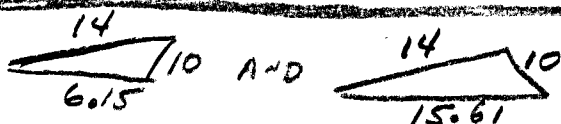
$$(-21.7588)^2 - 4(1)(96)$$

$$473.4454 - 384 = 89.45 = d$$

$$X = \frac{-b \pm \sqrt{d}}{2a} = \frac{+21.7588 \pm \sqrt{89.45}}{2}$$

$$X = \frac{21.7588 \pm 9.4578}{2}$$

$$X = \{15.61, 6.15\}$$



TWO SOLUTIONS

Summary - Solving Δ 's

If you have a Right Δ , use
the Pythagorean Theorem AND SOHCAHTOA
[$C^2 = a^2 + b^2$]

AS Needed.

NOT A Right Δ , use LOS, LOC

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

LOS

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C \end{aligned}}$$

LOC

↑
(Full of Angles)

Use For ASA,

AAS

(AAA)

NO NOT
A unique
 Δ

(Full of sides)

Use For SSS

SAS

(SSA)*

- use Cos (this angle)
- Solve QUADRATIC
- 0, 1, or 2 Δ 's

Homework: Solve each $\triangle$

① $A = 82^\circ$, $a = 9$, $b = 12$

② $A = 105^\circ$, $a = 18$, $b = 14$
