Exponential Functions

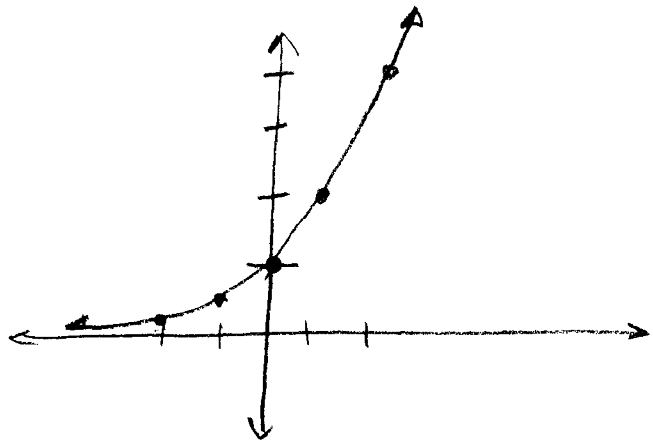
$y = A_0 b^x$

A_0 = starting value ($b^0 = 1$)
b = base

Ex $y = 2^x$

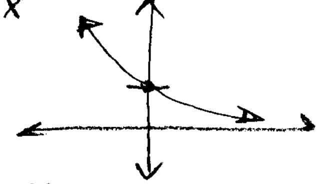
$A_0 = 1$ = starting value
base = 2

x	y
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	1
1	2
2	4



If base is >0 ⇒ exponential growth
If base is $0 < \text{base} < 1$ ⇒ exponential decay

$y = \left(\frac{1}{2}\right)^x$



Base ≠ negative (function would change from ± as x went even/odd)

Base ≠ 1 ⇒ function would be a horizontal line

The inverse of the EXPONENTIAL function (swap X and Y) $\Rightarrow$

$y = b^x$ EXPONENTIAL FUNCTION

$x = b^y$ inverse of exponential function

To get y "by itself" the inverse function is defined as the

$\boxed{\text{Logarithmic Function}}$

$$\log_b y = x$$

read "the logarithm of X with base b is y"

$$b^y = x$$

A logarithm is an exponent...
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Examples

Logarithmic Form	Exponential Form
$\log_2 16 = 4$	$2^4 = 16$
$\log_{10} 10 = 1$	$10^1 = 10$
$\log_3 1 = 0$	$3^0 = 1$
$\log_{10}(0.1) = -1$	$10^{-1} = \frac{1}{10} = .1$
$\log_2 6 \cong 2.585\ldots$	$2^{2.585} \approx 6$

$\log_3 9 = ?$

$3^x = 9$

$\boxed{x = 2}$

Base 10 = common log = no base shown

Ex $\log 14$ means $\log_{10} 14$

Base e = natural log = use ln

Ex $\ln 5$ means $\log_e 5$

$e \downarrow$ 2.71828...

Your calculator has the logs of base 10 and base e built-in.

To change or to find any base:

Change of Base Formula

$$\log_b X = \frac{\log_{10} X}{\log_{10} b} \quad \text{or} \quad \frac{\ln X}{\ln b}$$

Ex $$\log_3 14 = \frac{\log 14}{\log 3} = \frac{\ln 14}{\ln 3}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$= \frac{1.1461}{.4771} \quad \text{or} \quad \frac{2.6391}{1.0986}$$

$$= 2.4022 \quad \text{or} \quad 2.4022$$

CK: $\boxed{3^{2.4022} = 14.0004}$ ✓

Since logarithms are exponents, the exponent rules are related to the "logarithm" rules.

Exponent Rule	Log Rule
MR $b^m \cdot b^N = b^{m+N}$	$\log_b m + \log_b N = \log_b mN$
DR $\frac{b^m}{b^N} = b^{m-N}$	$\log_b m - \log_b N = \log_b \frac{m}{N}$
PPR $(b^m)^N = b^{m \cdot N}$	$\log_b m^N = N \log_b m$

Ex $\log_3 3 + \log_3 9 = \log_3 27$

$$\underbrace{3^x = 3}_{1} + \underbrace{3^x = 9}_{2} \stackrel{?}{=} \underbrace{3^x = 27}_{3} \checkmark$$

Ex $\log_3 9 - \log_3 3 = \log_3 \frac{9}{3} = \log_3 3$

$$\underbrace{\log_3 9}_{2} - \underbrace{\log_3 3}_{1} \stackrel{?}{=} \underbrace{\log_3 3}_{1} \checkmark$$

Ex $\log_3 3^4 = 4 \log_3 3$

$$4 \cdot \underbrace{3^x = 3}_{1} \stackrel{?}{=} 4 \checkmark$$

ACT PRACTICE

① What is the value of $\log_2 8$?

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② Find x:

$$\log_2 24 - \log_2 3 = \log_5 x$$

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③ Solve:

$$8^{2x+1} = 4^{1-x}$$

ANS

① $2^x = 8$ $\boxed{x = 3}$

or $2^x = 2^3$

② $\log_2 24 - \log_2 3 = \log_5 x$

$\log_2 \frac{24}{3} = \log_5 x$

$2^3 = 8$ $\log_2 8 = \log_5 x$

$3 = \log_5 x$ $\therefore$ $5^3 = x = \boxed{125}$

③ $8^{2x+1} = 4^{1-x}$

$2^{3(2x+1)} = 2^{2(1-x)}$ $\therefore$ $6x + 3 = 2 - 2x$

$8x = -1$

$\boxed{x = -\frac{1}{8}}$