

ACT
"REAL"
ACT

① THE RATIO OF THE RADII OF TWO CIRCLES IS 4:9 WHAT IS THE RATIO OF THEIR CIRCUMFERENCES

② IF 115% OF A NUMBER IS 460, WHAT IS 75% OF THE NUMBER?

③ IF $\log_a X = s$ AND $\log_a Y = t$ THEN $\log_a (XY)^2 = ?$

ANS

$$\textcircled{1} \frac{r_1}{r_2} = \frac{4}{9} \quad \begin{array}{l} C_1 = 2\pi r_1 \\ C_2 = 2\pi r_2 \end{array} = \frac{r_1}{r_2} = \boxed{\frac{4}{9}}$$

$$\textcircled{2} 1.15X = 460$$

$$X = \frac{460}{1.15} \quad \therefore \frac{460}{1.15} \left(\overset{.15}{.75} \right) = 2000(.15) = \boxed{300}$$

$$\textcircled{3} \log_a (XY)^2 = 2 [\log_a X + \log_a Y]$$

$$= \boxed{2[s + t]} \quad \text{or} \quad \boxed{2s + 2t}$$

Common Logarithm A logarithm with
a base of 10.

Normally the base
is left blank.

ⓔx $\log_{10} 100 = 2$ or $\log 100 = 2$

ck $10^2 = 100$ ✓

Ch. 10-4 Common Logarithms

The "log" key on a scientific
calculator is understood to be a
common logarithm.

NOTE: bring your scientific calculator
this week. Preferably your "ACT"
calculator. Recommended, any TI-30
flavor ≈ \$10.

The two "bases" built into scientific calculators are base 10 and base e

"COMMON"	≈ base 2.72
LOG key	"NATURAL"
LN key	

To solve logarithmic and exponential equations and get an actual number for the unknown variable, you are often going to take the common logarithm of both sides of the equation — usually stated as "take the log of both sides"

(for some advanced problems you will take the natural log of both sides, in this case include the word "natural" i.e. "take the NATURAL log of both sides")

EX 3
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Solve $3^x = 11$

NOTE: $3^2 = 9$
 $3^3 = 27$

Take the log of both sides

$$\log(3^x) = \log(11)$$

$$\frac{x \log 3}{\log 3} = \frac{\log 11}{\log 3}$$

$$x = \frac{\log_{10} 11}{\log_{10} 3} = \frac{\log_{10} 11}{\log_{10} 3}$$

$$x \approx \frac{1.04139}{0.47712} = 2.1827$$

$x \approx 2.1827$

CK $3^{(2.1827)} \stackrel{?}{\approx} 11.0005 \checkmark$

THIS IS ACTUALLY RELATED TO the "change" of base formula

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$$\log_b X = \frac{\log_{10} X}{\log_{10} b} = \frac{\ln X}{\ln b}$$

Common NATURAL

*M
log X ← top
b ← bottom
AND USE LOGS YOU KNOW

EX5
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Find $\log_4 25 = ?$

$$\log_4 25 = \frac{\log 25}{\log 4} \approx \frac{1.3979}{0.6021}$$

$\log_4 25 \approx 2.3217$

with
rounding
to 4th
decimal
place ≈ 2.3219

ck $4^{2.3217} \stackrel{?}{=} 24.9921$ ✓

$4^{2.3219} \stackrel{?}{=} 24.9990$ closer ✓✓

$$\textcircled{E} \text{ Solve } 2^{7x} = 3^{5x-3}$$

↑ ↑

NOTICE: UNLIKE BASES !!

TAKE log of BOTH SIDES

$$\log(2^{7x}) = \log(3^{5x-3})$$

$$\frac{7x \log 2}{(5x-3)} = \frac{(5x-3)(\log 3)}{(5x-3)}$$

$$\frac{7x}{5x-3} \log 2 = \log 3$$

$$\log 2 \qquad \log 2$$

$$\frac{7x}{5x-3} \approx 1.5850$$

$$7x = 1.585(5x-3)$$

$$7x = 7.925x - 4.755$$

$$+4.755 \quad -7x \qquad -7x + 4.755$$

$$\frac{4.755}{.925} = \frac{.925x}{.925}$$

$$\boxed{5.1405 \approx x}$$

$$\text{OK } 2^{7(5.1405)} \stackrel{?}{=} 3^{5(5.1405)-3}$$

$$2^{35.9835} \stackrel{?}{=} 3^{22.7025}$$

$$6.79 \times 10^{10} \stackrel{?}{=} 6.79 \times 10^{10} \checkmark$$

HOMEWORK: Pg 549 #4-7,9-11.