Alg. 2 BE

Thursday 5-10-12

① Which model is for "continuous" exponential growth/decay and which is for "non-continuous"?

$\boxed{y = a(1 \pm r)^t}$ or $\boxed{y = ae^{\pm rt}}$

② Which models each problem best, continuous or non-continuous exponential change?

ⓐ Investment of 5% compounded quarterly.

ⓑ Radioactive decay.

ⓒ Population change based on annual census.

ⓓ Investment of 5% compounded every minute.

Alg. 2 Homework Review - Pg. 563 #10, 11

(10) Computer = \$2500 Depreciates 20% per year
Value in 2 years? $\downarrow$ = 0.20 per year

$$C = C_0(1 \pm r)^t$$

$$C = 2500(1 - .20)^2 = 2500(.8)^2 = 2500(.64)$$

$\boxed{C = \$1600}$ = value of computer after 2 years

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(11) Condo = \$85,000 Value $\uparrow$ 5% per year
Value in 5 years = .05 per year

$$V = V_0(1 \pm r)^t$$

$$V = 85000(1 + .05)^5$$

$$V = 85000(1.05)^5 = 85000(1.2763\ldots)$$

$$\boxed{V = \$108{,}484}$$

Solving Exponential growth/decay Equations

EX1 PG 560

Cup of coffee = 130 mg caffeine

↓ Eliminated (body) ⇒ 11% = .11 per hr.

How long to eliminate half ?

C = mg of caffeine

$C = C_0(1-r)^t$ $\quad$ $t=0$ = drank 1 cup of coffee

$C = 130(1-.11)^t$ $\quad$ To eliminate $\frac{1}{2}$ $\Rightarrow \frac{130}{2} = 65$ mg

$65 = 130(.89)^t$ $\quad$ Solve for t

$\frac{65}{130} = (.89)^t$

$\frac{1}{2} = .89^t$ $\quad$ Take log or ln of both sides

$\ln\left(\frac{1}{2}\right) = \ln(.89)^t$

$\ln\left(\frac{1}{2}\right) = t\ln(.89)$

$\frac{\ln\left(\frac{1}{2}\right)}{\ln(.89)} = t$

$\therefore t = \frac{-.693147}{-.116534} = \boxed{5.9480 \text{ hours}}$

Ex2 RADIOACTIVE Decay
Pg 561 $\Rightarrow$ use $\boxed{A = A_0 e^{-rt}} = A = A_0 e^{-kt}$

Ⓐ Half-life $\Rightarrow$ time it takes for $\frac{1}{2}$ the ATOMS in A substance to decay.

$T_{1/2}$ of C-14 = 5760 years

FIND K for Carbon-14

$A = A_0 e^{-kt}$ AND $\frac{A}{A_0} = \frac{1}{2}$ if $t = 5760$ years

$\therefore \frac{1}{2} = e^{-K(5760)}$

$\ln\left(\frac{1}{2}\right) = \ln e^{-5760K}$

$\frac{-.693147}{-5760} = \frac{-5760K}{-5760}$

$\therefore K = -1.2034 \times 10^{-4}$

$\boxed{K = -.00012034}$

$\therefore$ MODEL THE DECAY of C-14

WITH $\boxed{A = A_0 e^{-.00012034t}}$ t = years

(b) Use your model for C-14 decay to estimate how long it takes for C-14 to reach 3% of starting value

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$$\Rightarrow \frac{A}{A_0} = .03 \quad \text{and} \quad A = A_0 e^{-.00012034\,t}$$

$$\therefore \quad .03 = e^{-.00012034\,t}$$

$$\ln(.03) = -.00012034\,t$$

$$\frac{\ln(.03)}{-.00012034} = t \qquad \text{where } t = \text{years}$$

$$\frac{-3.506557}{-.00012034} = t$$

$$\boxed{29138.76 \text{ years} = t}$$

Ex4
Pg 562

China ⇒ 1.26 billion = 1.26×10^9
India ⇒ 1.01 billion = 1.01×10^9

Models ⇒

$$C = 1.26e^{.009t}$$
$$I = 1.01e^{.015t}$$

When will $I > C$?

$$1.01e^{.015t} > 1.26e^{.009t}$$

$$\ln(1.01)(e^{.015t}) > \ln(1.26)(e^{.009t})$$

$$\therefore \ln(1.01) + \ln(e^{.015t}) > \ln(1.26) + \ln(e^{.009t})$$

$$\cancel{0.0099503} + .015t > .231112 + \cancel{.009t}$$
$$\cancel{-0.0099503} \quad -.009t \quad -0.0099503 \quad \cancel{-.009t}$$

$$\frac{.006t}{.006} > \frac{.221162}{.006}$$

$$\boxed{t > 36.86 \text{ years}}$$

Homework: Pg 564 #15–20