BE - Geometry 1

Thursday 1-5-12

ACT practice

① In the right $\triangle$ below $0 < b < a$. One of the angle measures in the $\triangle$ is $\tan^{-1}\left(\frac{a}{b}\right)$. What is $\cos\left[\tan^{-1}\left(\frac{a}{b}\right)\right]$?

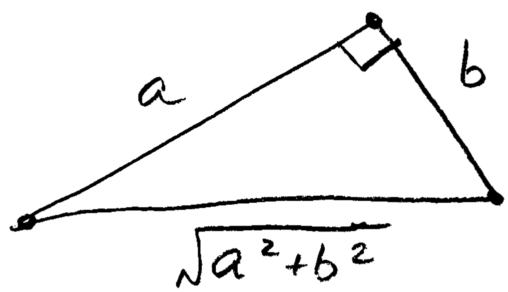
ⓐ $\frac{a}{b}$

ⓑ $\frac{b}{a}$

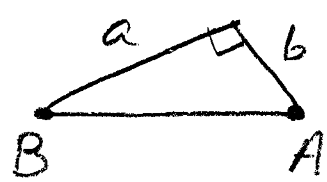
ⓒ $\frac{a}{\sqrt{a^2+b^2}}$

ⓓ $\frac{b}{\sqrt{a^2+b^2}}$

ⓔ $\frac{\sqrt{a^2+b^2}}{a}$



$\tan^{-1}\left(\frac{a}{b}\right)$ means the angle whose tan is $\frac{a}{b}$ which is Angle A.

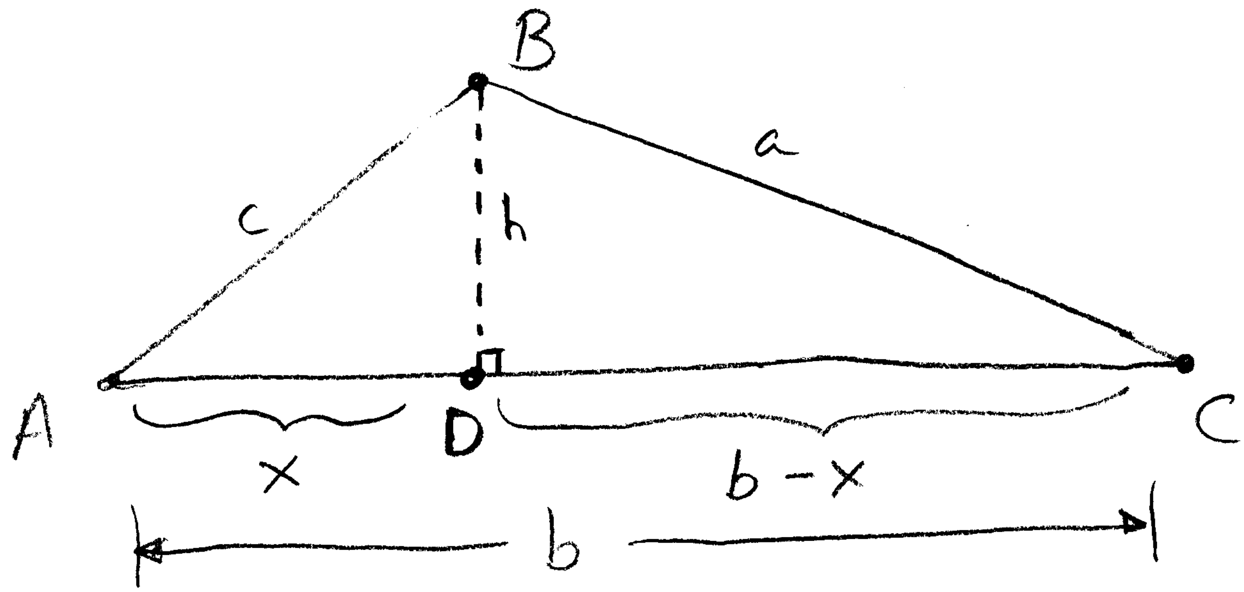


The $\cos A = \frac{b}{\sqrt{a^2+b^2}} = \frac{\text{adjacent}}{\text{hypotenuse}}$

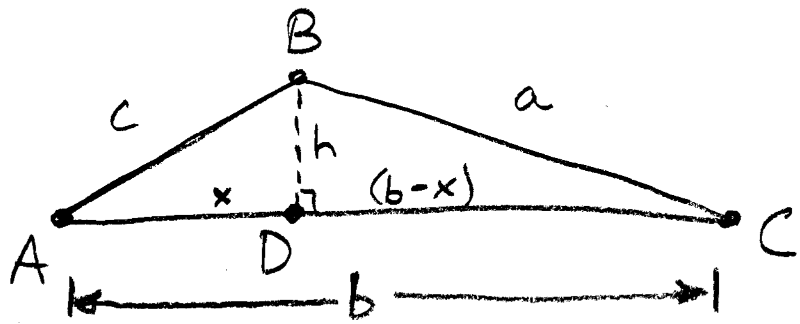
Answer d

There is another important triangle relationship – let's find it.

Take any $\triangle$ with standard labels and drop an altitude of length h. Let the 2 segments formed at B be x, $b-x$



Look at $\triangle BAD$ and $\triangle BCD$



$$a^2 = (b-x)^2 + h^2$$ P.T. IN $\triangle BDC$

$$\boxed{a^2 = b^2 - 2bx + x^2 + h^2}$$

$$x^2 + h^2 = c^2$$ P.T. IN $\triangle BAD$

$$\therefore\ a^2 = b^2 - 2bx + c^2$$ SUBSTITUTE

$$\boxed{a^2 = b^2 + c^2 - 2bx}$$

$$\cos A = \frac{x}{c} \quad \therefore \quad c\cos A = x$$

$$\therefore\ \boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$ LAW OF COSINES!

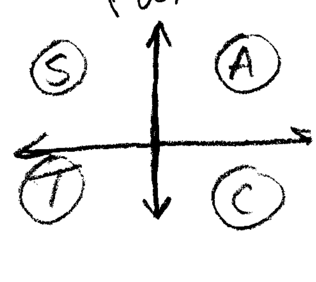
$$\underline{\underline{or}}\ \boxed{b^2 = a^2 + c^2 - 2ac\cos B}$$

$$\underline{\underline{or}}\ \boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

SIDE AND OPPOSITE Angle

WATCH OUT

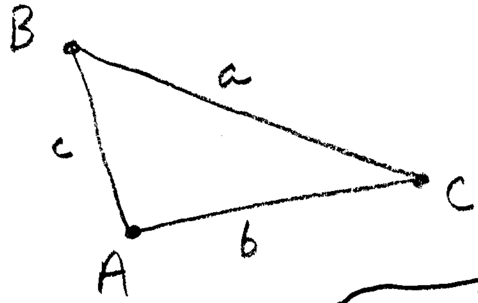
P.T.
if $C = 90^\circ$, $\cos 90^\circ = 0$



COSINE OF QUADRANT II Angles is NEGATIVE

LAW OF SINES

LAW OF COSINES



OPPOSITES

OPPOSITES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

* USE: 2 angles
  - ASA
  - AAS

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

* USE: 2 SIDES
  - SAS
  - SSS
  - * SSA   * SOLVE QUADRATIC Eq ⇒ 0, 1, or 2 (+) Solutions.

* The LOS and LOC are used when you do NOT have a Right △.

* If you DO have a right △, use
  - Sin, Cos, Tan
  - Pythagorean Theorem ⇒ $c^2 = a^2 + b^2$

EX 1
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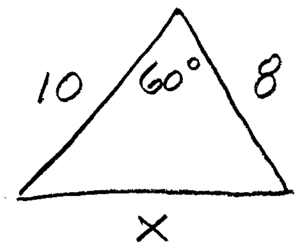
FIND X

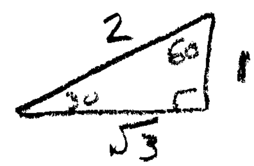


* {SAS, NOT RIGHT △, USE L of C.}

* WHICH FORM ?

$$x^2 = 10^2 + 8^2 - 2(10)(8)\cos 60^\circ$$

$$x^2 = 164 - 160\cos 60^\circ$$



$$x^2 = 164 - 160\left(\tfrac{1}{2}\right)$$

$$x^2 = 164 - 80$$

$$x = \sqrt{84}$$

4 21

$\therefore \boxed{x = 2\sqrt{21}}$

EXACT

$\boxed{x \approx 9.17}$

APPROXIMATE

* MUST USE FORM WITH COS(KNOWN ANGLE)

$$x^2 = \sim\sim \;\; \sim\sim \;\; \sim \cos 60^\circ$$

OPPOSITE SIDES

Ex 2
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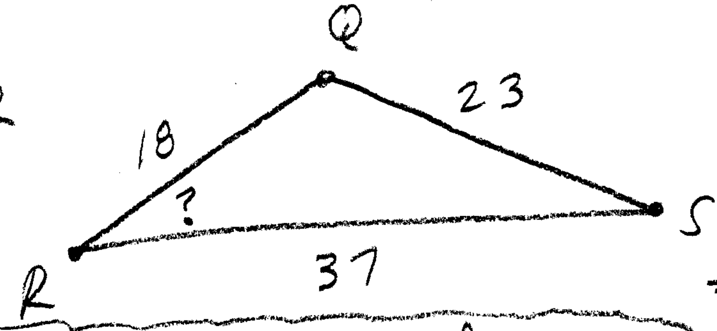
FIND $m\angle R$



* WHICH Form?

*SSS, NOT RIGHT $\triangle$, USE L. of Cosines

$$23^2 = 18^2 + 37^2 - 2(18)(37)\cos R$$

$$529 = 324 + 1369 - 1332\cos R$$

$$\frac{-1164}{-1332} = \frac{-1332\cos R}{-1332}$$

$$.8739 = \cos R$$

$$\boxed{\cos^{-1}(.8739) = 29.09^\circ}$$

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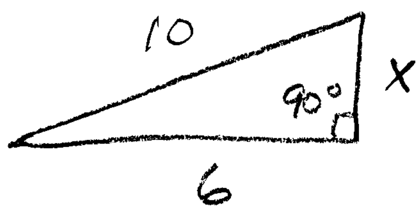
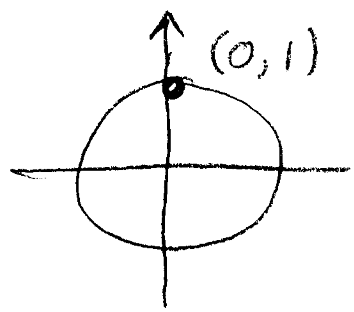
*CAN USE ANY OF THE 3 LOC forms, starting with largest angle/SIDE MAY be A good idea but I did NOT do that in this EXAMPLE.

WHAT HAPPENS if you accidently use LOC IN A right Δ.

Ex



$$10^2 = 6^2 + x^2 - 2(6)(x)\cos 90^\circ$$



$\cos 90^\circ = 0$

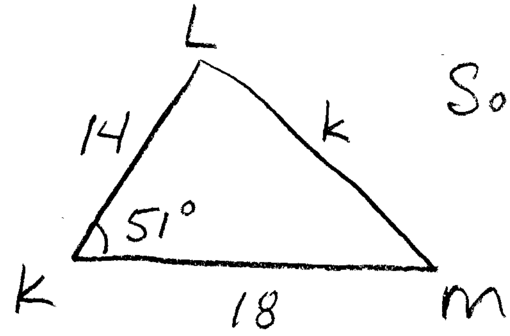
$\therefore\ 10^2 = 6^2 + x^2$ Pythagorean Theorem

$$x^2 = 100 - 36$$

$$x = 64$$

$$\boxed{x = 8}$$

Ex 3
Pg 386



Solve the $\triangle$

$\angle$s to nearest °
k to tenths

Find $m\angle L$, $m\angle M$, k

SAS $\Rightarrow$ LOC

$$k^2 = 14^2 + 18^2 - 2(14)(18)\cos 51°$$

$$k^2 = 196 + 324 - 504(.6293)$$

$$k^2 = 520 - 317.1774 = 202.8226$$

$$k = \sqrt{202.8266} \approx 14.24 \approx \boxed{14.2 = k}$$

* To find $m\angle L$ or $m\angle M$ you <u>COULD</u> use LOC, BUT THE LOS is much easier!

$$\frac{\sin 51}{14.2} = \frac{\sin L}{18} \quad \therefore \sin L = \frac{18 \sin 51}{14.2}$$

$$\boxed{\sin^{-1}(.9851) = 80° = m\angle L}$$

$$\therefore m\angle M = 180 - (51 + 80) = \boxed{49° = m\angle M}$$

Homework: Pg 388 # 20, 21