BE - Geometry 1

Friday 1-6-12

① Name the tools you would use to solve triangles with the following known sides or angles:

(A) ASA (B) SAS

(C) AAS (D) SSA

(E) SSS (F) AAA

② From memory, write formulas for:

(A) Pythagorean Theorem (B) Sin, Cos, Tan

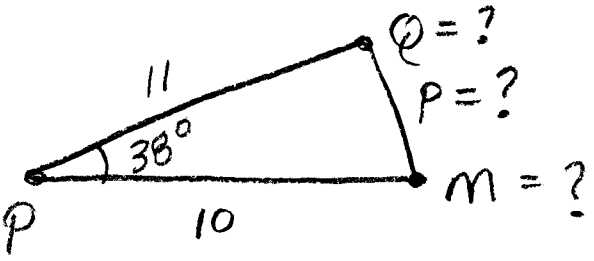
(C) Law of Sines (D) Law of Cosines

- Homework review

Homework Review Pg 388 # 20, 21

SOLVE:

(20)



∠s to nearest degree
sides to nearest tenth

USE THE LOC WITH KNOWN ANGLE P

$$p^2 = 11^2 + 10^2 - 2 \cdot 11 \cdot 10 \cdot \cos 38^\circ$$

$$p^2 = 121 + 100 - 220(.7880)$$

$$p^2 = 221 - 173.36 \quad \therefore p = \sqrt{47.64}$$

$$\boxed{p \approx 6.9}$$

$$\therefore \frac{\sin 38^\circ}{6.9} = \frac{\sin Q}{10}$$

$$Q = \sin^{-1}\left[\frac{10 \sin 38}{6.9}\right] = \sin^{-1}\left[\frac{6.157}{6.9}\right] = \sin^{-1}[.8923]$$

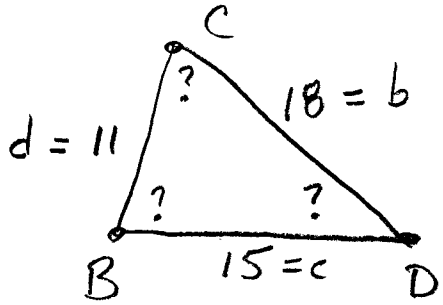
$$\boxed{Q \approx 63^\circ}$$

$$\therefore m = 180 - (38 + 63)$$

$$\boxed{m = 79^\circ}$$

Homework review

(21)



TIP: use LARGEST SIDE TO FIND LARGEST ∠ FIRST

$$b^2 = d^2 + c^2 - 2dc\cos B$$

$$18^2 = 11^2 + 15^2 - 2(11)(15)\cos B$$

$$-22 = -330\cos B$$

$$\therefore \cos B = \frac{22}{330} = .0667$$

$$\cos^{-1}(.0667) \approx \boxed{86^\circ = B}$$

$$\frac{\sin 86^\circ}{18} = \frac{\sin D}{11} \quad \therefore \sin D = \frac{11\sin 86^\circ}{18}$$

$$D = \sin^{-1}\left[\frac{11(.9976)}{18}\right]$$

$$D = \sin^{-1}[.6096]$$

$$\boxed{D \cong 38^\circ}$$

$$\therefore C = 180 - (38+86)$$

$$\boxed{C = 56^\circ}$$

The SSA case is different because for any SSA, there may be NO triangles, ONE triangle, or two triangles that exist with that particular SSA.

Most books call the the "Ambiguous" (unclear) Case of the Law of Sines And force you to memorize the conditions for each possibility. *(see next page) But there is, IMHO, A better way.

You can use the Law of Cosines, solve the resulting quadratic equation (EASY with a calculator) and let the ANSWER tell you WHAT you need to know.

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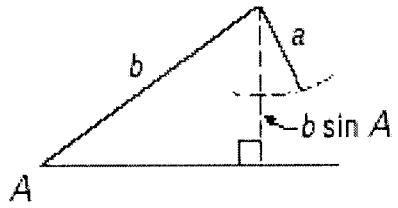
i.e., 0, 1, or 2 solutions. Here's how:

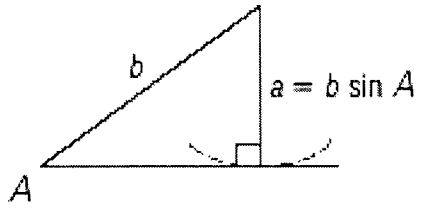
Key Concept — Possible Triangles Given Two Sides and One Opposite Angle

Suppose you are given a, b, and A for a triangle.

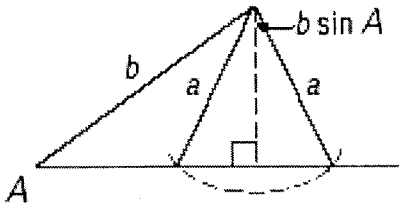
A Is Acute ($A < 90°$).



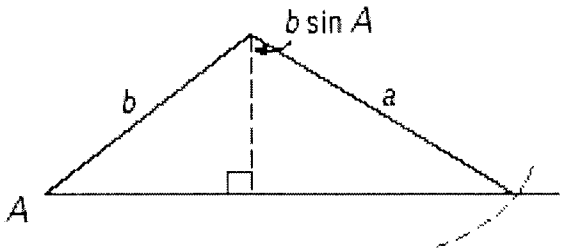
$a < b \sin A$
no solution



$a = b \sin A$
one solution

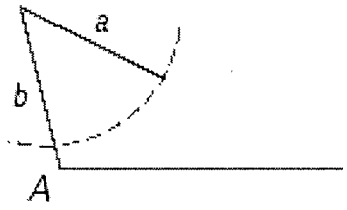


$b > a > b \sin A$
two solutions

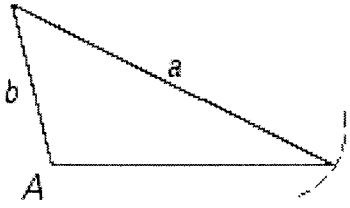


$a \geq b$
one solution

A Is Right or Obtuse ($A \geq 90°$).



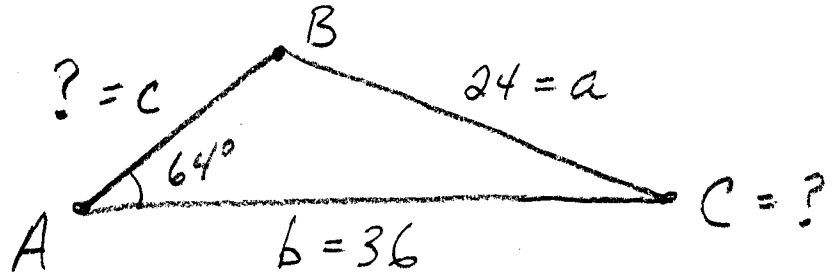
$a \leq b$
no solution



$a > b$
one solution

The Ambiguous Case of the Law of Sines = SSA ==> Use LOC and QE -- Mr.C.

(Ex) Solve: $a = 24 \quad b = 36 \quad A = 64°$



- SSA ⇒ Law of Cosines
- Use the known angle ⇒ $A = 64°$

$$24^2 = 36^2 + c^2 - 2 \cdot 36 \cdot c \cdot \cos 64°$$

$$576 - 1296 = c^2 - 72c \cdot (.4384)$$

$$-720 = c^2 - 31.5648c$$

$$\therefore \; c^2 - 31.5648c + 720 = 0$$

$a = 1$

$b = -31.5648$

$c = 720$

$b^2 - 4ac$

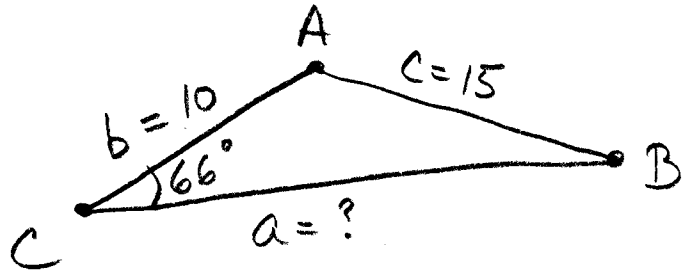
$(-31.5648)^2 - 4(1)720$

$996.3366 - 2880 = -1883.6634$

(-1883.6634 = discriminant)

Since d is negative, this triangle has no real solution

(EX) $b = 10,\ c = 15,\ C = 66^\circ$



$$15^2 = 10^2 + a^2 - 2 \cdot 10 \cdot a \cdot \cos 66^\circ$$

$$125 = a^2 - 20a(.4067)$$

$$a^2 - 8.134a - 125 = 0$$

$a = 1$ $\qquad b^2 - 4ac$

$b = -8.134$ $\qquad (-8.134)^2 - 4(1)(-125)$

$c = -125$ $\qquad 66.1620 + 500 = \boxed{566.1620 = d}$

$$a = \frac{-b \pm \sqrt{d}}{2a} = \frac{8.134 \pm \sqrt{566.1620}}{2}$$

$$a = \frac{8.134 \pm 23.794}{2}$$

NOT POSSIBLE

$\therefore \boxed{a = 15.964}$, -7.83

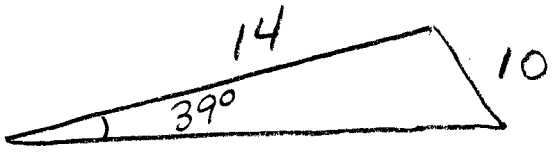
↑ ONLY possible ANSWER for a

FIND A, use LOS $\Rightarrow \dfrac{\sin A}{15.964} = \dfrac{\sin 66^\circ}{15}$

$$A = \sin^{-1}\left[\frac{15.964 \cdot \sin 66}{15}\right] = \sin^{-1}[.9722] \therefore \boxed{A = 76.5^\circ}$$

$$\therefore B = 180 - [66 + 76.5] = \boxed{37.5^\circ = B}$$

(Ex)



* SSA, ⇒ LOC ⇒ 0, 1, or 2 solutions possible

MUST use LOC Form LIKE THIS

$(\text{KNOWN SIDE})^2 = (\ )^2(\ )^2 - 2(\ )(\ )\cos(\text{KNOWN Angle})$

OPPOSITE

$10^2 = 14^2 + x^2 - 2(14)(x)\cos 39°$

$100 = 196 + x^2 - 28x(.7771)$

$0 = 96 + x^2 - 21.7588x$ QUADRATIC! USE QF

$x^2 - 21.7588x + 96 = 0$

$a = 1$

$b = -21.7588$

$c = 96$

$b^2 - 4ac$

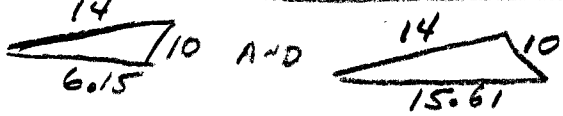
$(-21.7588)^2 - 4(1)(96)$

$473.4454 - 384 = 89.45 = d$

$$x = \frac{-b \pm \sqrt{d}}{2a} = \frac{+21.7588 \pm \sqrt{89.45}}{2}$$

$$x = \frac{21.7588 \pm 9.4578}{2}$$

$x = \{15.61, 6.15\}$ TWO SOLUTIONS



Summary – Solving $\triangle$'s

If you have a Right $\triangle$, use the Pythagorean Theorem and SOHCAHTOA

$$[c^2 = a^2 + b^2]$$

as needed.

Not a Right $\triangle$, use LOS, LOC

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LOS

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

LOC

↑

Full of Angles

Use for ASA, AAS

AAA

NO NOT a unique $\triangle$

Full of sides

Use for SSS

SAS

SSA *

- use Cos (this angle)
- Solve quadratic
- 0, 1, or 2 $\triangle$'s

Homework: Solve each $\triangle$

(1.) $A = 82^\circ$, $a = 9$, $b = 12$

(2) $A = 105^\circ$, $a = 18$, $b = 14$