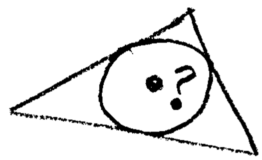
BE - Geometry !

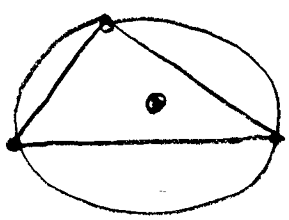
Monday 2-6-12

① What is the name of the center of the circle inscribed in a triangle?



② How do you find this center?

③ What is the name of the center of the circle circumscribed around a triangle?



④ How do you find this center?

⑤ Does $\csc^2\theta - \cot^2\theta = 1$?

ANS

① incenter ② intersection of Δ's $\angle$ bisectors.

③ circumcenter ④ intersection of Δ's perpendicular bisectors.

⑤ $\csc^2\theta - \cot^2\theta$

$$\frac{1}{\sin^2\theta} - \frac{\cos^2}{\sin^2\theta} = \frac{\boxed{1-\cos^2\theta}}{\sin^2\theta}$$

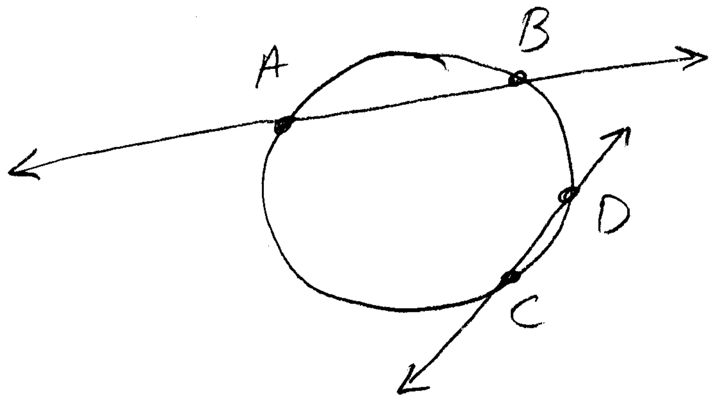
Using $\sin^2\theta + \cos^2\theta = 1$

$\boxed{\sin^2\theta = 1-\cos^2\theta}$

$$\Rightarrow \frac{\sin^2\theta}{\sin^2\theta} = 1 \checkmark$$

10-6 Secants, Tangents, and Angle Measures

A $\overleftrightarrow{\text{line}}$ that intersects a circle in exactly 2 points is called a **secant** Latin: "secare" to cut

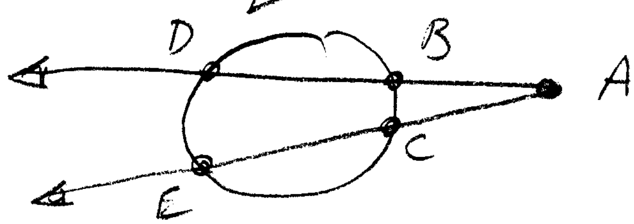


Line $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are secants.

Line segment $\overline{AB}$ and $\overline{CD}$ are *chords.

*A chord is a segment of a secant.

If two secants intersect at a point, they are also rays



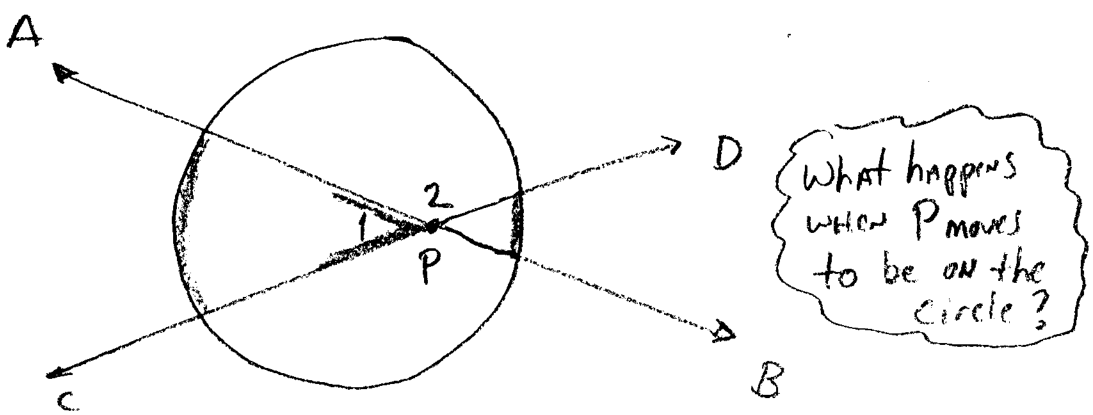
Ray $\overrightarrow{AB}$
Ray $\overrightarrow{AC}$
are secants.

THE "big" secant theory — memorize this... Really... you need to! IDI!

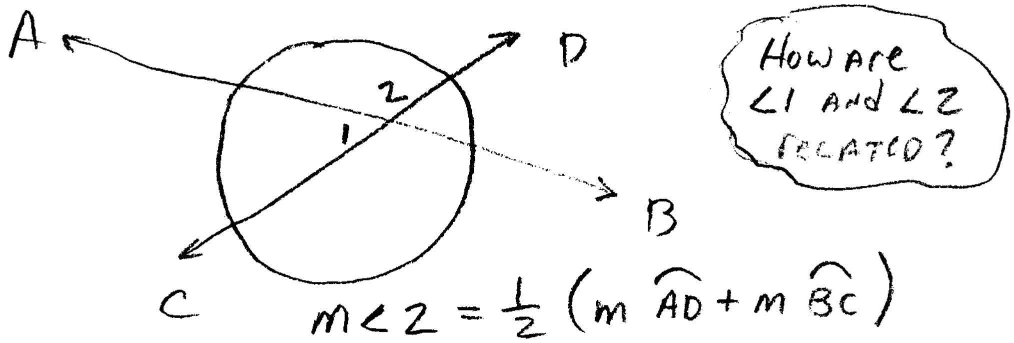
Theorem 10.12

If two secants intersect in the interior of a circle, then the measure of an angle formed is $\frac{1}{2}$ the <u>sum</u> of the measure of the intercepted arc and the arc of its vertical angle
(intercepted by)

Ex



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AC} + m\overset{\frown}{BD})$

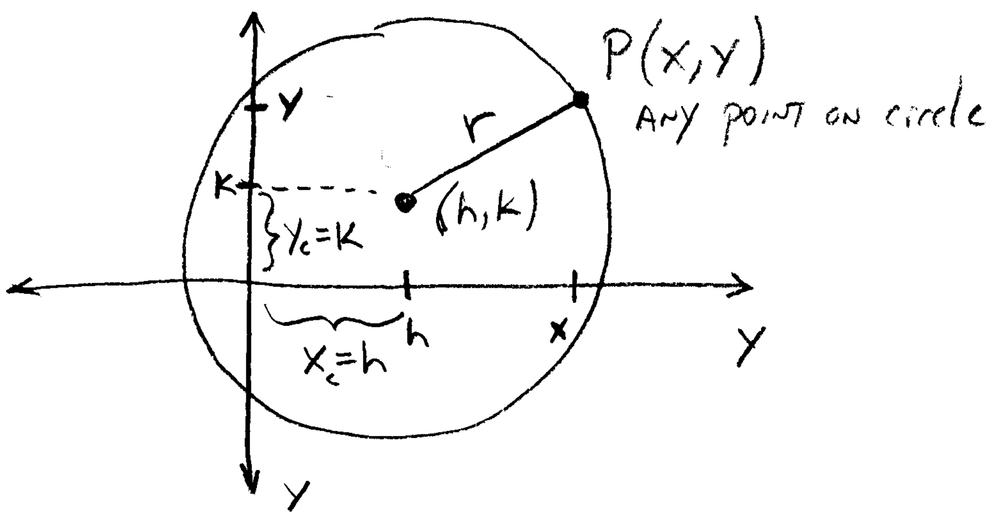


$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

Ch.10-8 Equations of Circles

Assume (x, y) are the coordinates of any point on the below circle. The center is at (h, k), the radius is r.

$$(\underset{x_c}{h}, \underset{y_c}{k})$$



Use the distance formula to find the distance from any P(x, y) to Center (h, k) which in a circle is the radius = r

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

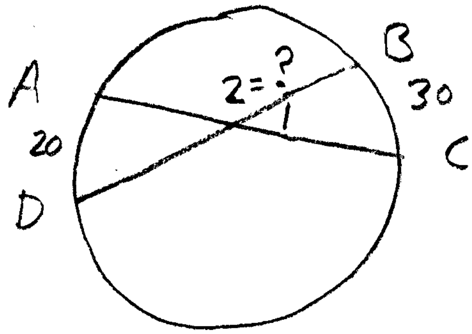
or $\boxed{(x-h)^2 + (y-k)^2 = r^2}$ Equation of circle with radius r and center (h, k).

Called the standard form of the Eq. of a ⊙

I prefer "center-radius" form

Ex 1
Pg 562

Find $m\angle 2$ if $m\overset{\frown}{BC} = 30$
$m\overset{\frown}{AD} = 20$



$$m\angle 1 = \frac{1}{2}(30 + 20) = 25^\circ$$

$$\therefore\ m\angle 2 = 180 - 25 = \boxed{155^\circ}$$

OR

Since $m\overset{\frown}{BC} + m\overset{\frown}{AD} = 50^\circ$

The $m\overset{\frown}{AB} + m\overset{\frown}{DC} = 310^\circ$

$$\therefore\ m\angle 2 \Rightarrow \frac{1}{2}(310) = \boxed{155^\circ}$$ ✓

There are many other relationships between secants, chords, and tangents of circles (see rest of 10-6, 10-7 for ex.)

Examples

$(x-3)^2 + (y-4)^2 = 4$ $\quad r = 2$

$(3,4) = \text{center}$

$(x-5)^2 + (y-1)^2 = 5$ $\quad r = \sqrt{5}$

$(5,1) = \text{center}$

$(x-7)^2 + y^2 = 9$ $\quad r = 3$

$(7,0) = \text{center}$

$x^2 + y^2 = 25$ $\quad r = 5$

$(0,0) = \text{center}$

$(x+2)^2 + (y-4)^2 = 10$ $\quad r = \sqrt{10}$

$(-2,4) = \text{center}$

Type into WolframAlpha:

circle (x+2)^2+(y-4)^2=10

A center-radius form can be put into general form: $\boxed{x^2+y^2+ax+by+c=0}$

General Form ⊙

Ex $(x-3)^2+(y-4)^2=4$

$x^2-6x+9+y^2-8y+16=4$

$x^2+y^2-6x-8y+21=0$

Can you go from general to center-radius form?

Yes, by completing the square. WATCH

$x^2+y^2-6x-8y+21=0$

Get x, y together, create holes for CTS

$x^2-6x+\{\ \}+y^2-8y+\{\ \}=-21+\{\ \}+\{\ \}$

½ "b" term squared

$x^2-6x+3^2+y^2-8y+4^2=-21+9+16$

$(x-3)^2+(y-4)^2=4$ ✓

In other words $\Rightarrow$ the center-radius form of the eq. of a ⊙

$$(x-h)^2 + (y-k)^2 = r^2$$

Expand to a perfect sq. trinomial

Expand to a perfect sq. trinomial

can be found by "completing the square" for x and y since this is how you create perfect square trinomials.

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* Follow the Golden Rule of Equations *

Homework: Pg 577 # 3, 4, 6, 7
Pg 578 # 10, 11, 12
Pg 564 # 12, 13, 14