

Ch. 12-3 Ellipses

- Very closely related to a circle, a circle can be thought of as a special case of an ellipse.
- Circle has one "focal point" so it is called the center. An ellipse has two focal points and a center.

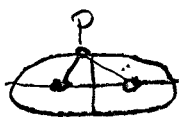
Notes: most books call this the midpoint of the ellipse but we will use center to match your book.

Geometry:



circle is the locus of all points equidistant from its focal point

Right-Left

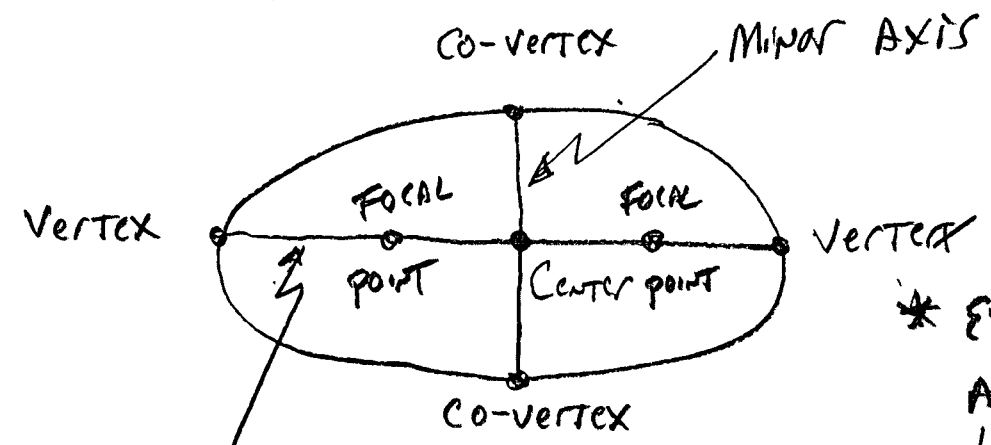


ellipse is the locus of all points such that the sum of the distances to the 2 focal points is constant

Up-Down



Naming the parts of an ellipse:

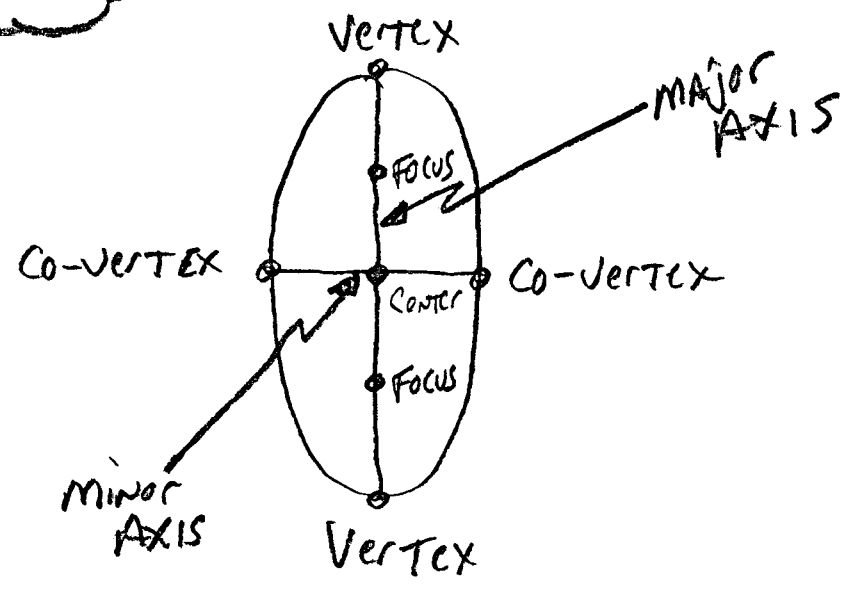


→ THINK OF THIS AS the minor RADIUS

* THINK OF THIS AS the major RADIUS

* EVEN THOUGH A ELLIPSE DOES NOT ACTUALLY HAVE A RADIUS

Left-Right Ellipse



UP-DOWN ELLIPSE

Comparing the Equations of Circles and Ellipses

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Divide by r^2 ,
the only radius

$$\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$$

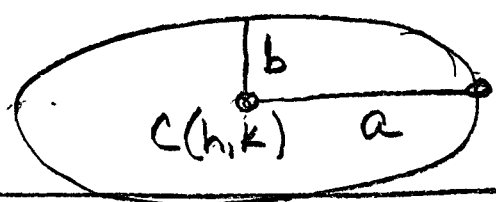
↑ SAME NUMBERS $r = \sqrt{r^2}$

Ellipse
(left-right)
BECAUSE NUMBER UNDER X term is bigger
↑ MAJOR AXIS

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

↑ DIFFERENT $a^2 > b^2$
↑ $a = \text{half of MAJOR AXIS } (r_{\text{major}})$
↑ $b = \text{half of MINOR AXIS } (r_{\text{minor}})$

NOTE: IF a^2 and b^2 are same you have a circle



Ellipse
(up-down)

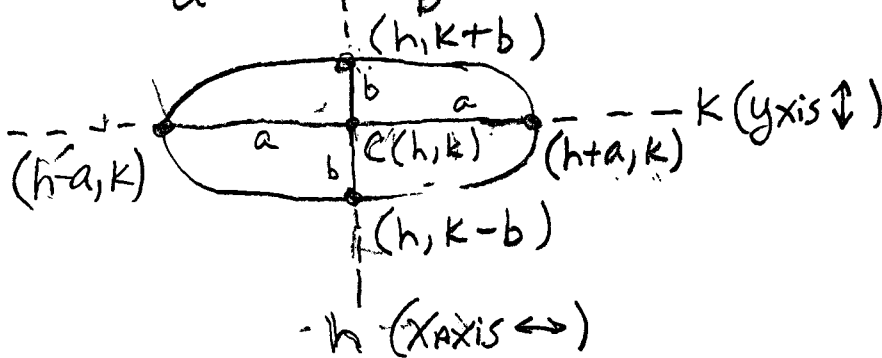
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$a^2 > b^2$,
"opens" up-down axis more,
∴ MAJOR AXIS IS up-down

Summary: How to find all the parts of an ellipse. (Pg 832 table)

Left-Right Ellipse (horizontal)

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad C(h,k)$$



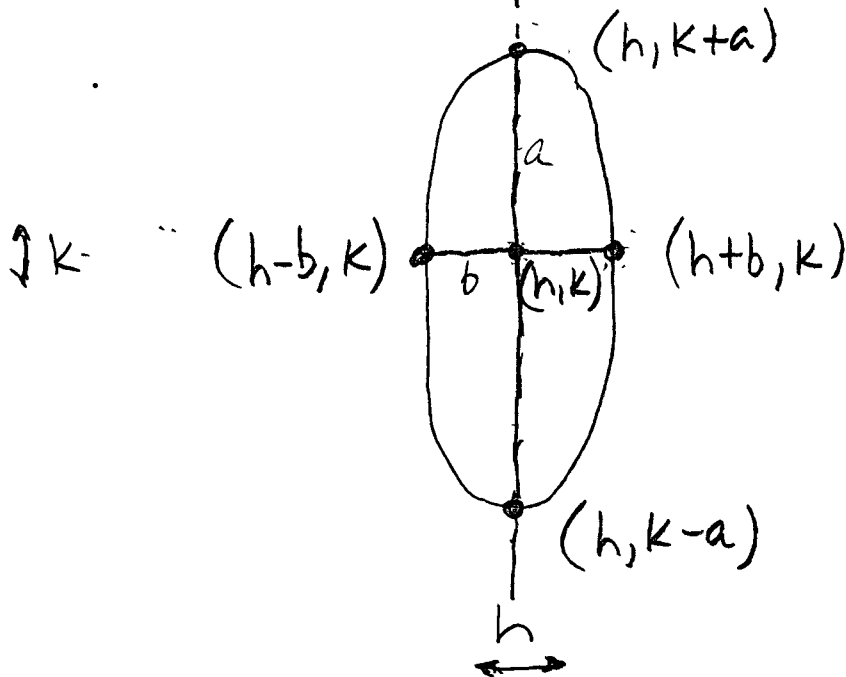
To get Foci, you need THIS FORMULA for ellipses:

c = memory AID

$$c^2 = a^2 - b^2$$

Then Foci, are at $(h+c, k)$ and $(h-c, k)$

Up-Down Ellipse (vertical)



$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$c^2 = a^2 - b^2$$

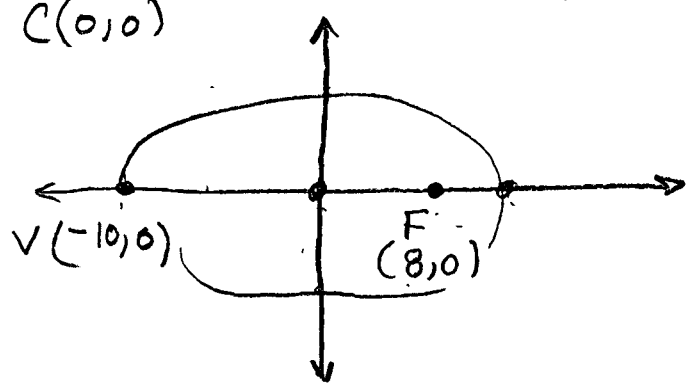
∴ Foci:

$$(h, k+c), (h, k-c)$$

FOCI ALWAYS ON MAJOR AXIS

EX 2
Pg 831

WRITE THE STANDARD FORM
OF the ellipse graphed below:



* Given Vertices
AND Foci
AND Center

Left-Right $\Rightarrow a = 10 \therefore a^2 = 100$
 $C(h,k) \Rightarrow (0,0)$
h k

$$\frac{(X-0)^2}{100} + \frac{(Y-0)^2}{b^2} = 1$$

but $c = 8 \therefore c^2 = 64$ AND $c^2 = a^2 - b^2$
Foci $\therefore 64 = 100 - b^2$

$b^2 = 36$
 $b = 6$

$\therefore \frac{X^2}{100} + \frac{Y^2}{36} = 1$ EOE

↑
 "major" radius = 10
 ↑
 "minor" radius = 6

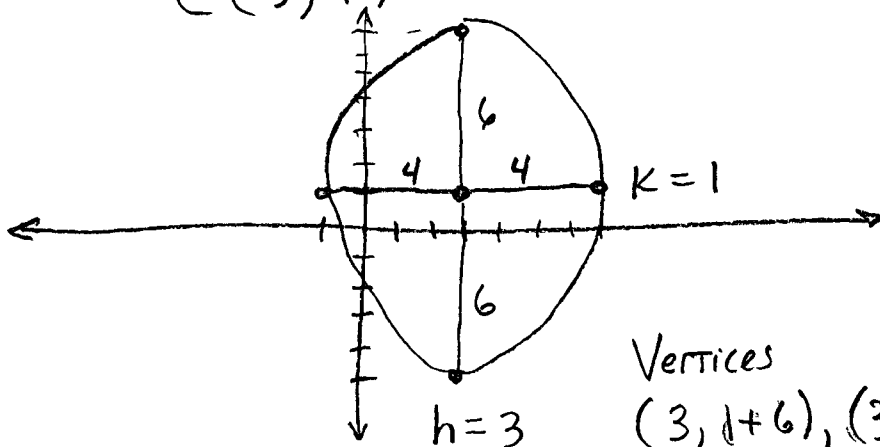
Ex 3
Pg 832

Graphing the ellipse:

$$\frac{(x-3)^2}{16} + \frac{(y-1)^2}{36} = 1$$

$b^2 \therefore b = 4$ "minor radius"
 $a^2 \therefore a = 6$, up-down ellipse
"major radius"

$C(3, 1)$



Vertices

$(3, 1+6), (3, 1-6)$

$V(3, 7), V(3, -5)$

Co-Vertices

$(3+4, 1), (3-4, 1)$

$CV(7, 1), CV(-1, 1)$

NOTE: the focal points
are NOT needed to graph
the ellipse as they are NOT
on the ellipse.

$$c^2 = a^2 - b^2$$

$$c^2 = 36 - 16 = 20$$

$$c = \sqrt{20} = 2\sqrt{5}$$

Foci $(3, 1+2\sqrt{5}), (3, 1-2\sqrt{5})$