

Ch. 12-7 Solving Nonlinear Systems (of equations)

Nonlinear system
of equations

A system of equations
in which at least one
of the equations is
non-linear.

(EX) $x^2 + y^2 = 25$ (Eq 1)
 $y + 5 = \frac{1}{2}x^2$ (Eq 2)

(Eq 1) = ?

circle

(Eq 2) = ?

parabola

How many solutions possible?



zero



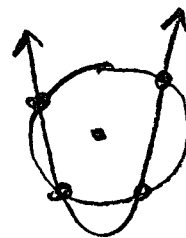
one



two



three



four

(Eg 1)

$$\begin{cases} x^2 + y^2 = 25 \\ 5 + y = \frac{x^2}{2} \Rightarrow 10 + 2y = x^2 \end{cases}$$

(Eg 2)

$$5 + y = \frac{x^2}{2} \Rightarrow 10 + 2y = x^2$$

EBA \Rightarrow line up "like" terms (as many as possible)

$$x^2 - 2y = 10 \longrightarrow x^2 - 2y = 10$$

$$x^2 + y^2 = 25 \xrightarrow{(-1)} -x^2 - y^2 = -25$$

$$-y^2 - 2y = -15$$

$$y^2 + 2y = 15$$

$$\therefore y^2 + 2y - 15 = 0$$

$$\text{sum} \Rightarrow b = 2$$

$$\text{prod} \Rightarrow ac = -15$$

$$-3 \quad +5$$

$$(y-3)(y+5) = 0$$

$$\therefore y = \{-5, 3\}$$

(Eg 1)

$$y = -5$$

$$\Rightarrow x^2 + (-5)^2 = 25$$

$$x^2 + 25 = 25$$

$$\therefore x = 0 \Rightarrow \boxed{(0, -5)}$$

(Eg 1)

$$y = 3$$

$$\Rightarrow x^2 + (3)^2 = 25$$

$$x^2 + 9 = 25$$

$$x^2 = 16$$

$$x = \pm 4 \text{ when } y = 3 \therefore$$

$$\boxed{(4, 3)}$$

$$\boxed{(-4, 3)}$$

$$\begin{cases} \text{Eq 1} & x^2 + y^2 = 25 \\ \text{Eq 2} & 5 + y = \frac{x^2}{2} \Rightarrow 10 + 2y = x^2 \end{cases}$$

EBS \Rightarrow get one "variable" by itself

$$\text{Eq 1} \Rightarrow (10 + 2y) + y^2 = 25$$

$$y^2 + 2y - 15 = 0$$

$$\text{sum} \Rightarrow b = 2$$

$$\text{prod} \Rightarrow ac = -15$$

$$\begin{array}{c} / \quad \backslash \\ +3 \quad +5 \end{array}$$

$$(y - 3)(y + 5) = 0 \quad \therefore y = \{-5, 3\}$$

\Rightarrow Same answers as EBA (see prior page)

Check with Wolfram Alpha

solving systems of equations



Examples Random

Assuming "solving systems of equations" refers to a computation | Use as a general topic instead

Assuming a system of two equations | Use instead

equation 1: $x^2 + y^2 = 25$

equation 2: $y + 5 = \frac{1}{2}x^2$

Input interpretation:

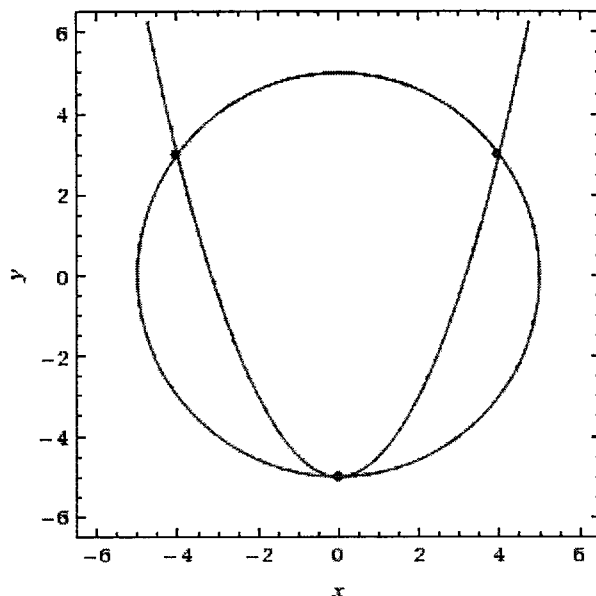
solve	$x^2 + y^2 = 25$
	$5 + y = \frac{x^2}{2}$

Results:

$x = \pm 4$ and $y = 3$

$x = 0$ and $y = -5$

Implicit plot:

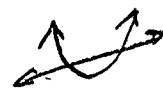


$x^2 + y^2 = 25$

$y + 5 = \frac{x^2}{2}$

$$\textcircled{2x} \begin{cases} 2x - y = 1 & \Rightarrow \text{LINE} \\ (y) + 7 = 2(x+1)^2 & \Rightarrow \text{PARABOLA} \end{cases}$$

$$\text{EBS} \Rightarrow y = \textcircled{2x-1}$$


0, 1, or two
SOLUTIONS

$$(2x-1) + 7 = 2[x^2 + 2x + 1]$$

$$\begin{array}{r} 2x + 6 = 2x^2 + 4x + 2 \\ -2x \quad -6 \qquad \qquad -2x \quad -6 \end{array}$$

$$2x^2 + 2x - 4 = 0$$

$$2(x^2 + x - 2) = 0$$

$$\text{sum} = b = 1$$

$$\text{prod} = ac = -2$$

$$2(x-1)(x+2) = 0 \quad \therefore x = \{1, -2\}$$

$$\textcircled{\text{Eg 1}} \quad x=1 \Rightarrow \begin{array}{l} 2(1) - y = 1 \\ -y = -1 \end{array} \quad \therefore y=1 \Rightarrow \boxed{(1, 1)}_{x \quad y}$$

$$\textcircled{\text{Eg 1}} \quad x=-2 \Rightarrow \begin{array}{l} 2(-2) - y = 1 \\ -4 - y = 1 \\ -5 = y \end{array} \quad \therefore y = -5 \Rightarrow \boxed{(-2, -5)}$$

How to check solutions?

systems of equations



Examples Random

Assuming "systems of equations" refers to a computation | Use as a general topic or referring to a mathematical definition instead

Assuming a system of two equations | Use instead

equation 1:

equation 2:

Input interpretation:

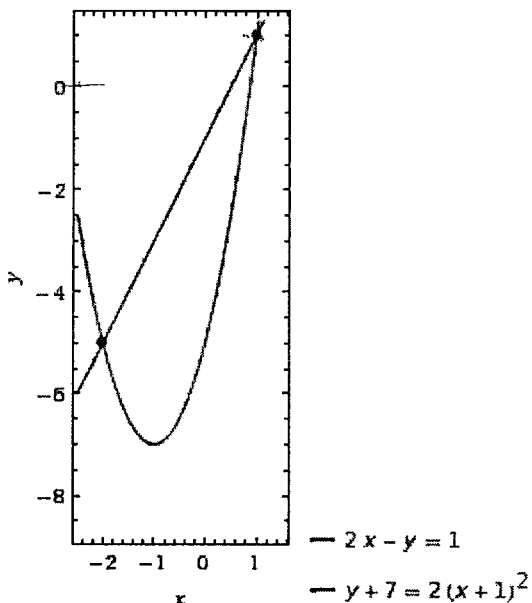
solve	$2x - y = 1$
	$7 + y = 2(1 + x)^2$

Results:

$x = -2$ and $y = -5$

$x = 1$ and $y = 1$

Implicit plot:



Worksheet Practice

$$\textcircled{1} \quad x^2 + 2y^2 + 2x + 4y - 17 = 0$$

$$\begin{aligned}
 & x^2 + 2x + \boxed{1^2} + 2y^2 + 4y + \boxed{} = 17 + \boxed{1+2} \\
 & \qquad \qquad \qquad + 2(y^2 + 2y + \boxed{1^2}) \\
 & \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow \\
 & \frac{(x+1)^2}{2} + \frac{2(y+1)^2}{2} = \frac{20}{2}
 \end{aligned}$$

$$\frac{(x+1)^2}{2} + \frac{(y+1)^2}{1} = 10$$

↪ A left-right ellipse

Still need
to get
right side
= to 1

$$\frac{(x+1)^2}{20} + \frac{(y+1)^2}{10} = 1$$

* Standard Form